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Strategic government subsidization for higher education in two-sector model

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Abstract

This paper examines strategic games of government and higher education. When we study strategic games of government and higher education, this game is then embedded in a two-stage game. The government chooses a subsidy to maximize social welfare and the two institutions choose the qualities simultaneously. The main results of the paper are to characterize the subgame-perfect equilibria of the government-higher education game, and to show that the governmental intervention may improve the social welfare by reducing the distortions due to externality and imperfect competition.

1 Introduction

This paper examines the incentive of government to provide subsidies to higher education. Educational finance is probably the most controversial issue in the economics of higher education. The historical development of higher education followed a different pattern than elementary and secondary schools. Higher education yields both private benefits to the student and public benefits to society at large. Governments subsidize higher education for the following reasons. Since investment in higher education is judged to bring economic and social benefits, one objective of government aid to higher education is to ensure that there is sufficient investment in higher education, and another is to promote equality of opportunity by ensuring that financial barriers do not prevent from enrolling in higher education.

There are some approaches to support higher education. For example, the government distributes the subsidy directly to the institutions, that is, colleges and universities. Other types of support for higher education are scholarships and tuition wavers, which are subsidies for higher education given directly to students. In practice, a majority of countries provide subsidies for higher education, but there are considerable differences between countries in the balance between aid to institutions and aid to students. The goal of government subsidization in the field of higher education naturally changes with the times.

One of the major motivations is to study the relationship between government and the higher education. While Fernandez and Rogerson (1995), [1] and Shirai and Furumatsu (1998)

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[1]Fernandez and Rogerson (1995) analyze that subsidies to higher education financed by general tax imply regressive income redistribution.

[2] analyze the subsidy for higher education given directly to students, in this paper we study the subsidy directly to the institutions.

In Japan, for example, the government is planning to subsidize the top 30 of higher education intensively. It has an aim to raise the standards of Japan's top 30 research universities to the world's highest level. It intends to do this through the infusion funding into five prioritized research areas. In the light of this, the question arises as to whether a university can operate effectively as a research-oriented establishment, or as a good teaching one.

The government intervenes in higher education in the form of subsidization, by which higher education manages successfully, together with using tuition fees, and so on. In this way, higher education is closely related to the government, so to speak, it is a game-theoretical situation. In general we tend to think of government intervention as a non-strategic activity. In our model, the government and higher education have their own objectives and interacts with each other.

There are two major assumptions in the paper. One is that our model is a sequential-move game. We clarify the differences between simultaneous-move game and sequential-move game. If the model is a simultaneous-move game, the two institutions must decide the quantities without knowing the subsidy provided by the government. On the other hand, if this is a sequential-move game, the two institutions observe the government subsidization and decide the quantities. In practice, it is difficult for the two institutions to decide the quantities without knowing the subsidy. Because Governments throughout the world provide financial support in higher education, and higher education cannot be managed without subsidy.

Another assumption is that we regard the institution of higher education as a profit-maximizing firm. According to studies by Nechyba (1996), [3] and Epple and Romano (1998), [4] it is assumed that a goal of the institution of higher education is to maximize profit.

Our approach is similar to the one found in Anant, Basu and Mukherji (1995). While their model is assumed to be a monopoly market and the government's objective is revenue-maximization, we assume that the government maximizes social welfare and the two institutions maximize their individual profits in a heterogeneous duopoly market. While the consumers in group 1 and 2 demand the educational services, [5] the two institutions provide a differentiated product in the sense that one is teaching-oriented product and another is research-oriented product.

In this paper, we present three main results. First, the main results of the paper are to characterize subgame-perfect equilibria of government-higher education game. Second, the social welfare may be improved by using the optimal subsidy system, which reduces the distortions due to externality and imperfect competition. Finally, we show that if they are practiced on

[2] Shirai and Furumatsu (1998) point out that subsidies should be given directly to qualified students rather than to educational institutes.

[3] Nechyba (1996) considers a three-community model with public and private schools, migration, voting over expenditure level, and peer effects. The private sector is relatively passive with regard to peer effects because each private school can only charge one price, and therefore can only attract one type of student. He finds that as a result of a voucher policy, school based stratification increase, but residential stratification decrease. This is because parents who send their children to private schools migrate to find communities with lower tax rates.

[4] Epple and Romano (1998) consider a model with students differing continuously over ability and income. The education production function that they use depends on the average ability of the students in the school and the student's ability, but not on the expenditures of the school. They find that under a voucher system the achievement gains of those high ability students who switch from the public school to the private school, and hence a better peer group are great, while the loses of the left behind are small. However the number of students failing into the later category is much greater. Their model also yields monotonically increasing welfare gains in the size of the voucher.

[5] In general, we allowed to enter the institution of higher education by the entrance examination. But in this paper, whether they are admitted to enter either institution is distinguished a piori by the two different types.

the basis of the top 30 scheme, these results are contrary to the subsidization policy proposed.

This paper is organized as follows: In section 2 the model is outlined. In the first period, the government chooses the subsidy to maximize social welfare and in the second period, the two institutions choose the qualities simultaneously. We solve for the equilibria at each stage in the model by backward induction. The equilibrium concept that we use is a subgame-perfect Nash equilibrium. In Section 3 to understand the effects of subsidy, we compare the two subsidies. These results are contrary to the subsidization policy of the top 30 scheme. Finally in Section 4 is for conclusion and a discussion of some further studies.

2 The model

The institutions of higher education are extremely complex organizations and seek to achieve a multiplicity of goals simultaneously. For example, some universities emphasize the provision of comprehensive liberal education and others intend to conduct the most advanced research. In Japan, it may be expected that private universities tend to specialize in the provision of undergraduate training, while public universities tend to have a broader set of goals objectives. In setting up this model, we do not distinguish between public higher education and private higher education. [6] Hence, substantial variations among institutions may be found in the priorities assigned to goals and objectives (the typical contrast is that between teaching and research.)

2.1 Households

Consider an economy with two types of representative consumers. While a model with two types was studied by de Bartolome (1990) and Benabou (1996), [7] in this paper we assume the following situation. They faced with two higher institutions' options, providing higher education of teaching-oriented and research-oriented. The two types, 1 and 2, attend either institution teaching-oriented or research-oriented depending upon their preferences. We assume that type 1 attends a teaching-oriented institution and type 2 attends a research-oriented institution.

We assume the existence of two representative consumers is postulated whose preferences over consumption of the two products (q_T, q_R) and the numeraire good $z_i (i = 1, 2)$ take the quasilinear form below:

$$U^i(q_T, q_R, z_i) = \alpha_i q_j - \frac{1}{2}\beta_i q_j^2 + \gamma q_T q_R + z_i. \ (i = 1, 2 \quad j = T, R) \tag{1}$$

[6] In Japan, the government is planning to change the national university system so as to reduce the size of government organization and to make it more efficient. The national universities are turned into the Independent Administrative Institutions by the year 2003. If this is carried out, it makes no difference between public and private higher education.

[7] de Bartolome (1990) constructs an environment with two types individuals, two types of communities, and schools where peer effects matter. Because his school system is public, the peer effect is not priced, and is therefore an externality. Benabou (1996) extends this model. He includes a capital market, and derives several conditions under which the communities are stratified, and when this is efficient. Depending upon the specification of the production function for human capital, he finds that a policy to equalize school budgets could either lead to integration or cause no change in mixing across the segregated communities. If communities remain segregated, the poor are better off due to the policy, but the rich are much worse off, implying a reduction in average achievement. Community integration could lead to an increase in average achievement.

where α_i and $\beta_i (i = 1, 2)$ are all positive. The parameter γ is the external effect. Normalize the price of the numeraire to be unity. The market demand functions are derived by the utility-maximizing behavior of the two representative consumers given the prices and the after tax income:

$$p_j q_j + z_i = \bar{m}_i. \ (i = 1, 2 \quad j = T, R)$$

The utility maximization problem is

$$max. \quad U^i(q_T, q_R, z_i) = \alpha_i q_j - \frac{1}{2}\beta_i q_j^2 + \gamma q_T q_R + z_i.$$

$$subject\ to. \quad p_j q_j + z_i = \bar{m}_i.$$

Take the derivatives with respect to q_j. Then we obtain the inverse demand functions, respectively, as

$$p_T = \alpha_1 + \gamma q_R - \beta_1 q_T \tag{2}$$

and

$$p_R = \alpha_2 + \gamma q_T - \beta_2 q_R. \tag{3}$$

2.2 Educational Sector

In educational sector, there are two types of educational institutions; the institution of teaching-oriented higher education and the institution of research-oriented higher education.

Teaching institution is denoted by T, and research institution is denoted by R. Each institution produces the qualities q_T and q_R respectively, which are measured by hours per period of faculty time in various disciplines, hours per period of non-faculty labor, hours of utilization per period of laboratory equipment, hours of students' time, the number of BAs(MAs,PhDs) in each of the various disciplines per period, and the number of research of those disciplines per period.

Two institutions independently and simultaneously decide how much to produce q_T and q_R. We suppose that the cost functions of the institutions $j (j = T, R)$ are $c_j(q_j) = c_j q_j$. Then the institutions' profits are given by

$$\begin{aligned} \pi^T(q_T, q_R) &= p_T q_T - c_T q_T + s_T q_T \\ &= (\alpha_1 + \gamma q_R - \beta_1 q_T - c_T + s_T) q_T \end{aligned}$$

and

$$\begin{aligned} \pi^R(q_T, q_R) &= p_R q_R - c_R q_R + s_R q_R \\ &= (\alpha_2 + \gamma q_T - \beta_2 q_R - c_R + s_R) q_R. \end{aligned}$$

The first-order conditions for the institutions T and R are

$$\frac{\partial \pi^T(q_T, q_R)}{\partial q_T} \quad : \quad \alpha_1 + \gamma q_R - 2\beta_1 q_T - c_T + s_T = 0 \tag{4}$$

and

$$\frac{\partial \pi^R(q_T, q_R)}{\partial q_R} \quad : \quad \alpha_2 + \gamma q_T - 2\beta_2 q_R - c_R + s_R = 0. \tag{5}$$

From (4) and (5), we obtain the Cournot-Nash equilibrium outputs in terms of the subsidy system s_T and s_R:

$$q_T(s_T, s_R) = \frac{2\beta_2 s_T + \gamma s_R + 2\beta_2(\alpha_1 - c_T) + \gamma(\alpha_2 - c_R)}{4\beta_1\beta_2 - \gamma^2} \tag{6}$$

and

$$q_R(s_T, s_R) = \frac{\gamma s_T + 2\beta_1 s_R + \gamma(\alpha_1 - c_T) + 2\beta_1(\alpha_2 - c_R)}{4\beta_1\beta_2 - \gamma^2}. \tag{7}$$

2.3 Government

The government distributes subsidies directly to two institutions. The subsidies are financed by lump-sum tax imposed on consumers. One objective of government aid to higher education is to ensure that there is sufficient investment in higher education, and another is to promote equality of opportunity by ensuring that financial barriers do not prevent from enrolling in higher education. In this paper the government chooses the subsidy of the two institutions to maximize social welfare.

$$\begin{aligned} W &= U^1 + U^2 \\ &= (\alpha_1 - c_T)q_T + (\alpha_2 - c_R)q_R + 2\gamma q_T q_R - \frac{1}{2}\beta_1 q_T^2 - \frac{1}{2}\beta_2 q_R^2 + \bar{z}. \end{aligned}$$

where $\bar{z} = \sum \bar{z}_i (i = 1, 2)$. The last equation is derived by using the demand functions.

2.4 Equilibrium

We consider the following two-stage game:

1. Government decide subsidies s_j $\quad (j = \mathrm{T,R})$
2. Each institution q_j simultaneously decides q_j $\quad (j = \mathrm{T,R})$

The aim of this section is to show equilibrium of strategic games of government and the institutions of higher education. In period 1, the government chooses the subsidy. In period 2, with the subsidy given, two institutions choose the individual qualities. The equilibrium concept that we use is a subgame-perfect Nash equilibrium.

The strategy profile $(s_T^*, s_R^*, q_T^*(s_T^*, s_R^*), q_R^*(s_T^*, s_R^*))$ is a subgame-perfect equilibrium:

$$W(s_T^*, s_R^*) \geq W(s_T, s_R^*) \qquad for\ all\ s_T$$

and

$$W(s_T^*, s_R^*) \geq W(s_T^*, s_R) \qquad for\ all\ s_R$$

and

$$\pi^T(q_T^*(s_T^*, s_R^*), q_R^*(s_T^*, s_R^*)) \geq \pi^T(q_T(s_T, s_R), q_R^*(s_T^*, s_R^*)) \qquad for\ all\ q_T$$

and

$$\pi^R(q_T^*(s_T^*, s_R^*), q_R^*(s_T^*, s_R^*)) \geq \pi^R(q_T^*(s_R^*, s_R^*), q_R(s_T, s_R)) \qquad for\ all\ q_R.$$

We now solve the first stage of the game. Taking into account the social welfare function and two institutions' reactions to the subsidy, the government determines the welfare-maximizing subsidy. Given the welfare function, we now calculate the optimal subsidy system s_T^* and s_R^*:(cf. Appendix)

$$s_T^* = \frac{(2\gamma^2 + \beta_1\beta_2)(\alpha_1 - c_T) + 3\beta_1\gamma(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2} \tag{8}$$

and

$$s_R^* = \frac{3\beta_2\gamma(\alpha_1 - c_T) + (2\gamma^2 + \beta_1\beta_2)(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2}. \tag{9}$$

Substituting (8) and (9) into (6) and (7) yields (cf. Appendix)

$$q_T^* = \frac{\beta_2(\alpha_1 - c_T) + 2\gamma(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2} \tag{10}$$

and

$$q_R^* = \frac{2\gamma(\alpha_1 - c_T) + \beta_1(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2}. \tag{11}$$

The equilibrium output $q_T^* > 0$ and $q_R^* > 0$ require that $\alpha_1 > c_T$, $\alpha_2 > c_R$ and $\gamma > 0$. We first consider the conditions that $\alpha_1 > c_T$ and $\alpha_2 > c_R$. These conditions imply that the institution provides positive q_l when $q_j = 0 \quad (j, l = T, R \quad l \neq j)$. Assuming that $\alpha_1 > c_T$ and $\alpha_2 > c_R$, then the institution j always chooses to enter the market of higher education. The condition $\gamma > 0$ implies that higher educations bring the external benefit for each other. Furthermore, if q_T^*, and q_R^* are positive, then (8) and (9) are strictly positive.

3 Policy implication

The object of this section is to examine the characteristics of the optimal subsidy system. We now focus on the terms s_j.

Proposition 1. *When the government-higher education game are played, the price are lower than the marginal cost, i.e.* $p_T < c_T$ *and* $p_R < c_R$ *respectively.*

Proof. Substituting (2) and (3) into (4) and (5), we obtain

$$p_T - \beta_1 q_T - c_T + s_T = 0, \tag{12}$$

and

$$p_R - \beta_2 q_R - c_R + s_R = 0. \tag{13}$$

Hence, the equations (12) and (13) are rewritten as:

$$\begin{aligned} p_T - c_T &= \beta_1 q_T - s_T \\ &= \frac{-\gamma[2\gamma(\alpha_1 - c_T) + \beta_1(\alpha_2 - c_R)]}{\beta_1\beta_2 - 4\gamma^2} \\ &< 0 \end{aligned}$$

and

$$\begin{aligned} p_R - c_R &= \beta_2 q_R - s_R \\ &= \frac{-\gamma[\beta_2(\alpha_1 - c_T) + 2\gamma(\alpha_2 - c_R)]}{\beta_1\beta_2 - 4\gamma^2} \\ &< 0. \end{aligned}$$

Then we obtain Proposition 1.

This result is not surprising, because the optimal subsidy system should be introduced to improve the distortions due to the imperfect competition and the externality. When $s_j = \beta_i q_j$ in equation (12) and (13), we obtain $p_j = c_j$, which means the social production efficiency is attained. But the distortion by the externality still remains in addition to the distortion of imperfect competition. It is necessary to correct the distortion of the externality by the difference. The characteristics of the subsidy system are also given in Proposition 2.

Proposition 2. *(1) The larger the price elasticity of the demand is, the smaller the optimal subsidy is. The smaller the price elasticity of the demand is, the larger the optimal subsidy is.*
(2) The larger the complemental consumption is, the larger the optimal subsidy is. The smaller the complemental consumption is, the smaller the optimal subsidy is.

Proof. Note that the equations (8) and (9). From (8) and (9), we obtain $s_j = \beta_i q_j + \gamma q_l \quad (i = 1, 2 \quad j, l = T, R \quad j \neq l)$. The optimal subsidy system is divided into the two terms.

The value of the first term on the right-hand side is the distortion of imperfect competition. Without the external effect, the demand functions are denoted by $\alpha_i - \beta_i q_j = p_j$. Therefore, the terms $\beta_i q_j$ are rewritten as

$$\beta_i q_j = \frac{p_j}{q_j}\frac{dq_j}{dp_j} = \frac{p_j}{e_j}.$$

where e_j is the price elasticity of the demand. Hence, we obtain Proposition 2 (1).

The value of the second term on the right-hand side is the marginal external benefit from (1). Hence, we obtain Proposition 2 (2).

We next turn to an analysis of the effect of the subsidies. To do so, we compare the amounts of two subsidies. The difference between s_T and s_R is

$$s_T^* - s_R^* = \frac{1}{\beta_1\beta_2 - 4\gamma^2}\Big[(\alpha_1 - c_T)(2\gamma^2 - 3\beta_2\gamma + \beta_1\beta_2) - (\alpha_2 - c_R)(2\gamma^2 - 3\beta_1\gamma + \beta_1\beta_2)\Big].$$

Proposition 3. *When the optimal subsidies s_T^* and s_R^* are compared, the relative amounts of the optimal subsidies s_T^* and s_R^* are determined by the condition (14).*

$$s_T^* \gtreqless s_R^* \quad \leftrightarrow \quad \frac{(\alpha_1 - c_T)(2\gamma^2 - 3\beta_2\gamma + \beta_1\beta_2)}{(\alpha_2 - c_R)(2\gamma^2 - 3\beta_1\gamma + \beta_1\beta_2)} \gtreqless 1. \tag{14}$$

This comparison between the optimal subsidies reveals an interesting point regarding the role of the government. Several features of proposition 3 deserve comment.

First, suppose that $\alpha_1 = \alpha_2$, and $\beta_1 = \beta_2$. If $c_T < c_R$, then $s_T^* > s_R^*$. In practice, research is a very time-consuming exercise and costly. We can regard c_R as the marginal cost of the sector of research-oriented higher education. Then it means that the government provides the more subsidies for the sector of teaching-oriented higher education than the sector of research-oriented higher education.

Second, now suppose that $\alpha_1 = \alpha_2$ and $c_T = c_R$. If $\beta_1 > \beta_2$, then $s_T^* > s_R^*$. The parameters β_1 and β_2 measure the degree of the diminishing marginal utility. We can regard β_2 as the sector of research-oriented higher education. Thus it means that the government provides the more subsidies for the sector of teaching-oriented higher education than the sector of research-oriented higher education.

These results are contrary to the subsidization policy proposed. The ultimate goal of the top 30 scheme is to elevate Japanese research universities to the apex of international excellence. To practice this policy the government intends to be prioritized research areas. But in this paper there is some possibility that the distortion is expanded by this policy.

4 Concluding remarks

In this paper, we introduce the strategic government subsidization in the field of higher education and find that it has important implications regarding the welfare effects of the subsidy. When the optimal subsidy system is used, they may improve the distortions due to externality and imperfect competition. But, if they are practiced on the basis of the top 30 scheme, these results are contrary to the subsidization policy proposed.

Economists' views on higher education are quite diverse and economic studies on higher education have been increasing in numbers in recent years. But some basic questions on the economic roles of higher education still remain unanswered. One of the unsolved problems is what subsidization policies should be in the field of higher education. In this paper, we study this subsidiary problem in view of the game-theoretical analysis.

The social environment for higher education is expected to have made great changes from the current state by the beginning of the 21st century. Higher education has become increasingly important as a means of maintaining Japan's social vitality and improving national living standards. The government recognizes the important role played by higher education and is working to expand fiscal support for national local and private universities and other institutions of higher education.

Important question we have ignored in our analysis are how the present analysis is affected by considering individual decisions of educational services taking account of labor supply and asset holdings. The analysis in the labor market and asset market is undoubtedly complex. These and other possible improvement of this paper is topics for future research.

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Appendix

Deriving equilibrium properties (8)-(11)

First, the demand function is derived from the utility function. The utility maximization problem is

$$max. \quad U^i(q_T, q_R, z_i) = \alpha_i q_j - \frac{1}{2}\beta_i q_j^2 + \gamma q_T q_R + z_i. \quad (i = 1, 2) \tag{15}$$

$$subject\ to. \quad p_j q_j + z_i = \bar{m}. \quad (j = T, R) \tag{16}$$

Substituting (16) into (15) we obtain

$$max. \quad U(q_1, q_2, z) = \alpha_i q_j - \frac{1}{2}\beta_i q_j^2 + \gamma q_T q_R + \bar{m} - p_j q_j. \tag{17}$$

Take the derivatives with respect to q_j. Then the first-order conditions for the utility maximization are:

$$\alpha_1 + \gamma q_R - \beta_1 q_T - p_T = 0 \tag{18}$$

and

$$\alpha_2 + \gamma q_T - \beta_2 q_R - p_R = 0. \tag{19}$$

From (18) and (19) we obtain the inverse demand functions, respectively, as

$$p_T = \alpha_1 + \gamma q_R - \beta_1 q_T$$

and

$$p_R = \alpha_2 + \gamma q_T - \beta_2 q_R.$$

Next, we obtain the quantities q_T and q_R by maximizing institutions' profit. The institutions' profits are given by

$$\begin{aligned}\pi^T(q_T, q_R) &= p_T q_T - c_T q_T + s_T q_T \\ &= (\alpha_1 + \gamma q_R - \beta_1 q_T - c_T + s_T) q_T\end{aligned}$$

and

$$\begin{aligned}\pi^R(q_T, q_R) &= p_R q_R - c_R q_R + s_R q_R \\ &= (\alpha_2 + \gamma q_T - \beta_2 q_R - c_R + s_R) q_R.\end{aligned}$$

The first-order conditions for the institutions T and R are

$$\frac{\partial \pi^T(q_T, q_R)}{\partial q_T} \; : \; \alpha_1 + \gamma q_R - 2\beta_1 q_T - c_T + s_T = 0 \tag{20}$$

and

$$\frac{\partial \pi^R(q_T, q_R)}{\partial q_R} \; : \; \alpha_2 + \gamma q_T - 2\beta_2 q_R - c_R + s_R = 0. \tag{21}$$

From (20) and (21), we obtain the Cournot-Nash equilibrium outputs in terms of the subsidy system s_T and s_R:

$$q_T(s_T, s_R) = \frac{2\beta_2 s_T + \gamma s_R + 2\beta_2(\alpha_1 - c_T) + \gamma(\alpha_2 - c_R)}{4\beta_1\beta_2 - \gamma^2} \tag{22}$$

and

$$q_R(s_T, s_R) = \frac{\gamma s_T + 2\beta_1 s_R + \gamma(\alpha_1 - c_T) + 2\beta_1(\alpha_2 - c_R)}{4\beta_1\beta_2 - \gamma^2}. \tag{23}$$

The one thing that remains to be shown is that social welfare function: The welfare is represented by

$$\begin{aligned} W &= U^1 + U^2 \\ &= (\alpha_1 - c_T)q_T + (\alpha_2 - c_R)q_R + 2\gamma q_T q_R - \frac{1}{2}\beta_1 q_T^2 - \frac{1}{2}\beta_2 q_R^2 + \bar{z}. \end{aligned}$$

The social welfare maximization problem is

$$\max_{s_T, s_R} W$$

$$s.t. \quad S = s_T q_T + s_R q_R.$$

Hence, we obtain the optimal subsidy system s_T and s_R:

$$s_T^* = \frac{(2\gamma^2 + \beta_1\beta_2)(\alpha_1 - c_T) + 3\beta_1\gamma(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2} \tag{24}$$

and

$$s_R^* = \frac{3\beta_2\gamma(\alpha_1 - c_T) + (2\gamma^2 + \beta_1\beta_2)(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2}. \tag{25}$$

Substituting (24) and (25) into (22) and (23) yields

$$q_T^* = \frac{\beta_2(\alpha_1 - c_T) + 2\gamma(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2} \tag{26}$$

and

$$q_R^* = \frac{2\gamma(\alpha_1 - c_T) + \beta_1(\alpha_2 - c_R)}{\beta_1\beta_2 - 4\gamma^2}. \tag{27}$$