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# Innovation through Global Collaboration in an Endogenous Growth Model*

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**Abstract**

This note investigates how innovation takes place in a global economy where research and development (R&D) firms in different countries are engaged in a single innovation project. By extending a standard R&D-based growth model, the note identifies two major factors of innovation through global collaboration: (i) Whether the global innovation takes place through collaboration or outsourcing depends on the relative productivity between global and local innovation. If the global collaboration is made through collaboration, (ii) the rate of global innovation depends on the relative endowment of skilled labor between two countries.

JEL Classifications: F43, O30
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# 1 Introduction

The international collaboration has become increasingly important for many companies that work on research and development (R&D). As MacCormack et al. (2007a, b) note, “innovation through global collaboration is becoming a new source of innovation and knowledge creation in the global economy.” An important example is Boeing’s 787 Dreamliner aircraft, which was developed by more than 50 partners in over 130 locations.[1]

One key aspect of this trend is that “collaboration is not outsourcing” (MacCormack et al. 2007a). Outsourcing typically refers to the use of substituting partners as a means to achieve lower costs through wage arbitrage. A firm engaging in global collaboration may intend to go beyond simple wage arbitrage, blending its unique expertise “with skills from other partners developing complementary technologies.” Although outsourcing of innovation has been investigated by Lai, Riezman, and Wang (2009), neither innovation through global collaboration nor the explicit distinction of collaboration and outsourcing has been formally addressed in the literature on endogenous innovation. The present note intends to take the first step towards filling this void.

For this purpose, I extend the R&D-based growth model developed by Romer (1990) by incorporating global innovation activities that internationally fragmented partners invent new technologies. In accordance with those arguments, I distinguish, by definition, global innovation through collaboration and that through outsourcing by how complementary internationally fragmented partners involved in a global innovation activity are. Naturally, I relate a complementary relationship between international partners to collaboration, and a more substituting relationship to outsourcing.[2]

Using this model, I reveal that how global innovation takes place in the market depends on two key economic parameters. (i) Whether the global innovation takes place through collaboration or outsourcing depends on the relative productivity between global and local innovation. If the global collaboration is made through collaboration, (ii) the rate of global innovation

[1] Another example is the creation of a joint innovative solutions lab, called OZONE, in 2008 by Oracle in California and Wipro in Bangalore. See MacCormack et al. (2007a) for more examples.

[2] This view is consistent with the vision of Grossman and Rossi-Hansberg (2008). In their model of outsourcing in production, foreign and local tasks (and thus labors) are assumed to be perfect substitutes.

increases the foreign endowment of skilled labor but decreases with the home endowment of skilled labor.

# 2 A Model of Global Innovation and Economic Growth

I will incorporate global collaboration activities to develop innovations into Romer's growth model (Romer, 1990).[3] Consider two countries: the home country (Country $H$) and the foreign country (Country $F$). These countries are basically symmetric and integrated both financially and through trade, differing only in their endowments of labor and knowledge stocks.

In each country, the representative consumer who inelastically supplies skilled and unskilled labor. Assume that skilled labor can be also used as unskilled labor, which cannot play a role as skilled labor. The home country supplies $R^H$ units of skilled labor and $L^H$ units of unskilled labor; the foreign country supplies $R^F$ units of skilled labor and $L^F$ units of unskilled labor. Skilled labor is used for invention while unskilled labor is used for production of goods.

There are a number of differentiated consumption goods distributed on interval $[0, N_t]$, where $N_t$ denotes the number of the goods. The space of goods expands with endogenous innovations, each consisting of a new production technology for manufacturing a new consumption good. Each good, say good $j$, is produced by a manufacturing firm that holds the exclusive licence on production technology for good $j$. The license on the invented good, $j$, is traded between good $j$'s inventor and a particular manufacturing firms in the international licensing market. The produced units of good $j$ are traded between the particular manufacturing firm holding the license and consumers in the international consumption good market.

## 2.1 Global R&D

There are a number of competitive R&D firms in the home country. The R&D firm can invent a new good in each of the following two ways. The first

[3] The model presented below is based on the model of globally collaborative innovation developed by Furukawa (2008). We extend his analysis by introducing a clear distinction between collaboration and offshoring as channels of global innovation.

way is inventing with home and foreign labor. In order for a firm to make an invention in period $t$, it is necessary to employ $h_t^H$ units of home labor and $h_t^F$ units of foreign labor in period $t$. We call this way of making an invention "global innovation." Assume that the elasticity of substitution between home skilled labor and foreign skilled labor is constant at $\varepsilon > 0$; the R&D firm's technology can take the form of a constant elasticity of substitution function.

$$1 = \left(\psi^G\right)^{-1} \left[ \left(A_t^H h_t^H\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(A_t^F h_t^F\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $\psi^G > 0$ represents the cost of global innovation and $A_t^i$ the cumulative stock of knowledge capital in region $i$. The firm chooses $h_t^H$ and $h_t^F$ so as to minimize the cost. Solving the standard optimization problem, we can write the (unit) cost function for a global innovation as

$$c_t = \psi^G \left[ \left(w_{R,t}^H / A_t^H\right)^{1-\varepsilon} + \left(w_{R,t}^F / A_t^F\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

where $w_{R,t}^i$ denotes the wage rate for Country $i$'s skilled labor.

Elasticity of substitution $\varepsilon$ captures the explicit distinction between global innovation through collaboration and that through offshore outsourcing. As is well known, if $\varepsilon < 1$, home labor $h_t^H$ and foreign labor $h_t^F$ are complementary in the sense that the firm increases (decreases) the ratio of home and foreign labor, $h_t^H / h_t^F$, less than proportionally as the ratio of home and foreign wages, $w_{R,t}^H / w_{R,t}^F$, decreases (increases). This implies that the job assigned to foreign labor $h_t^F$ is highly differentiated with the job for home labor $h_t^H$.[4] In accordance with the discussion in the introduction based on MacCormack et al. (2007a, b), I may define collaboration by complementary $h_t^H$ and $h_t^F$. Accordingly, and in line with the vision of Grossman and Rossi-Hansberg (2008), I may relate outsourcing to substitutable resources.[5] If $\varepsilon > 1$, home labor $h_t^H$ and foreign labor $h_t^F$ are said to be substitutable in the sense that the firm increases the ratio $h_t^H / h_t^F$ more than proportionally as $w_{R,t}^H / w_{R,t}^F$ decreases. This implies that the job for foreign labor is qualitatively similar to that for home labor.[6] Consequently, in this note, global innovation is said

[4] In this case, $h_t^H$ and $h_t^F$ are both essential for a research project that the R&D firm works on.

[5] Grossman and Rossi-Hansberg (2008) assume that foreign and local labors (i.e., tasks) are "perfect" substitutes in production of goods. See, for instance, Chu, Cozzi, and Furukawa (2015) for an application of such a setting to an endogenous growth model.

[6] Note that, in this case, the firm tends to hire more labor whose wage is relatively low.

to be done through collaboration if $\varepsilon < 1$ and through outsourcing if $\varepsilon > 1$.[7]

Turn to the second way for the firm to invent a new good by using only home skilled labor. We call this process of inventing a new good "local innovation." Assume that firm can invent a new product through local innovation by hiring $l_t = \psi^L / A_t^H$ units of home skilled labor. Here $\psi^L$ represents the cost of of local innovation. The unit cost on home local innovation is given by

$$d_t = \frac{\psi^L w_{R,t}^H}{A_t^H}. \tag{3}$$

Since the output invention of each of these two types of innovation is identical, we can denote the value of innovation by a single expression that is independent of the types of innovation, say $V_t$. The R&D firm that invents a new good, $j$, earns the value $V_t$ by selling the exclusive license on good $j$ to a particular manufacturing firm in the international license market. Given $V_t$, the firm optimally chooses the type of innovation whose cost is lower than or equal to that of the other type of innovation. Specifically, if there exists only an R&D firm investing in local innovation, it must hold that $c_t \geq d_t$; its payoff is $V_t - d_t$. If there exists only an R&D firm investing in global innovation, it must hold that $c_t < d_t$; the payoff is $V_t - c_t$. If an R&D firm investing in global innovation and an R&D firm investing in local innovation coexist in the marketplace, it must hold $c_t = d_t$. The R&D firm's net profit can be summarized by $V_t - \min\{c_t, d_t\}$.

We exclude a possibility in which the foreign county is engaged in local R&D, by implicitly assuming $\psi^L$ of the foreign country is sufficiently large. That is, it is too costly to invest in local R&D in the foreign country. This implies that the home country is permanently technologically advanced relative to the foreign country. Taking a look at reality and history, this should

[7] To understand the role of $\varepsilon$ intuitively, think of a simplified example. There is an aircraft maker. It hires home and foreign labor from home and foreign countries to develop a new airplane. Suppose that the maker develops an innovation through collaboration. This implies that it assigns highly differentiated (i.e., complementary) jobs to home and foreign labor in the basic stage of innovation (e.g., development of a new engine and that of a new body). In this case, the maker would not tend to change the ratio of home and foreign workers largely even if the wage ratio drastically changes. Suppose that the maker develops an innovation through outsourcing. This implies that it assigns similar jobs to home and foreign labor (e.g., safety tests or simple paperworks that anyone can do in so much the same way). In this case, the maker would tend to increase foreign labor (and decrease home labor) drastically as the wage rate of foreign labor decreases, so as to lower the cost taking an opportunity of international wage arbitrage.

not be realistic; however, given that the main purpose of this paper is to figure out mechanics of innovation through global collaboration, we assume $\psi^L$ for the foreign country is sufficiently high. Recently, Furukawa (2015) develops a new growth model with an infinitely lived agent where technological leadership endogenously shifts between countries along an equilibrium path.

## 2.2 Manufacturing

A manufacturing firm buying the exclusive licence on good $j$ from the inventing R&D firm at date $t$ produces $x_t\left(j\right)$ units of good $j$ at date $t$ and subsequent dates. It can earn a sequence of profits by monopolistically selling the produced goods to consumers in the international good market. Assume that the manufacturing firm freely chooses where the good is produced, the discounted sum of profits depends. Denote by $W_t^i\left(j\right)$ the discounted sum of profits for the manufacturing firm that produces in Country $i$, where $i = H$ or $i = F$. The location choice problem for the manufacturing firm, $j$, is represented by

$$\max_{i=\{H,F\}} W_t^i\left(j\right) = \int_t^\infty e^{-\int_t^\tau r_s ds} \pi_\tau^i\left(j\right) d\tau, \tag{4}$$

where $r_t$ is the world interest rate and $\pi_t^i\left(j\right)$ represents the profit of the manufacturing firm that locates in Country $i$.

The manufacturing firm that locates in Country $H$ can produce one unit of any consumption good by using one unit of domestic unskilled labor while the firm locating in Country $F$ must use $z$ units of foreign unskilled labor. The temporary profit $\pi_t^i\left(j\right)$ of manufacturing firm $j$ locating in Country $i$ should solve the following problem. Subject to the derived market demand function (which is endogenously given later),

$$\max_{\{x_t^i(j), p_t(j)\}} \pi_t^i(j) = p_t^i(j)x_t^i(j) - \Gamma(i)w_t^i x_t^i(j) \tag{5}$$

where $w_t^i$ represents the wage rate for unskilled labor in Country $i$ and $\Gamma$ satisfies $\Gamma(H) = 1$ and $\Gamma(F) = z$.

## 2.3 Consumption

The demand structure of the model is standard. The representative consumer exists in each country who is endowed with the following intertemporal utility function:

$$U=\int_0^{\infty} e^{-\rho t}\ln u_t dt. \tag{6}$$

The utility $u_t$ is defined as a constant elasticity of substitution utility function on $N_t$ differentiated consumption goods:

$$u_t=\left(\int_0^{N_t} x_t(j)^{(\sigma-1)/\sigma}dj\right)^{\sigma/(\sigma-1)}, \tag{7}$$

where $\sigma>1$ is the elasticity of substitution between any two goods.

## 2.4 Market Equilibrium

To complete the model description, I will write on the conditions for market equilibrium. Denote as $N_t^G$ and $N_t^H$ the numbers of goods developed by, respectively, global innovation and home innovation.; $N_t=N_t^G+N_t^H$. Assume $N_0^G\geq 0$ and $N_0^H>0$.

Using these numbers of goods,I can first express the labor market clearing conditions. Denote as $\Lambda_t^i$ the set of goods that are manufactured in Country $i$. Note that $\Lambda_t^H\cap\Lambda_t^F=\emptyset$. and that $\Lambda_t^H\cup\Lambda_t^F$ is equivalent to the set of all available goods that are invented up to date $t$. Then the labor market conditions are described as follows:

$$R^H=\dot{N}_t^G h_t^H+\dot{N}_t^H l_t \text{ and } L^H=\int_{j\in\Lambda_t^H} x_t(j)\,dj \tag{8}$$

for the home country;

$$R^F=\dot{N}_t^G h_t^F \text{ and } L^H=\int_{j\in\Lambda_t^F} x_t(j)\,dj \tag{9}$$

for the foreign country.

To close the model, I consider three non-arbitrage conditions. The first condition is for the locational choice, which requires that each firm manufacturing firm, $j$, is indifferent to the country: $W_t^H(j)=W_t^F(j)$. Since the goods (and the firms) are symmetric on $j$, the non-arbitrage condition for location can be written as

$$W_t^H(j)=W_t^F(j)=W_t(j), \tag{10}$$

where manufacturing firms locating in Countries $H$ and $F$ share the single expression on the discounted sum of profits, $W_t\left(j\right)$.

The second and third non-arbitrage conditions emerge from the assumption of free entry into the international license market. The free entry assumption must require zero profit for both sides of the market. On the buyer's side, a potential manufacturing firm (licensee) can freely enter into the market. Since the licensed manufacturing firm's net payoff is equal to $W_t\left(j\right) - V_t$, the following non-arbitrage condition must hold.

$$W_t\left(j\right) = V_t. \tag{11}$$

On the seller's side, a potential R&D firm can freely enter into the market to earn the value of innovation, $V_t$, by inventing a new good. As I already mentioned, the R&D firm's net payoff is equal to $V_t - \min\left\{c_t, d_t\right\}$. The following non-arbitrage condition holds.

$$V_t = \min\left\{c_t, d_t\right\}. \tag{12}$$

This completes the description of the model and its equilibrium.

# 3 Determinants of Innovation through Global Collaboration

Recall that parameters $\psi^G$ and $R^F$, respectively, represent how costly global innovation activities are and how abundant the endowment of skilled labor is in the foreign country. In this section, I will demonstrate how these parameters affect the condition under which, and the extent to which, global innovation occurs through collaboration.

First, I examine the role of the cost of global innovation, $\psi^G$. Recall that R&D firms invest in global innovations only if

$$c_t \leq d_t. \tag{13}$$

Before proceeding, it is useful to denote as $\omega_t$ the international inequality of skilled-labor wage measured in terms of efficiency unit:

$$\omega_t = \frac{w_{R,t}^H / A_t^H}{w_{R,t}^F / A_t^F}.$$

This may be said as the quality-adjusted price of home skilled labor relative to foreign skilled labor. By (2) and (3), (13) can be rewritten as

$$\left(\psi^G/\psi^L\right)\left(1+(\omega_t)^{-(1-\varepsilon)}\right)^{\frac{1}{1-\varepsilon}} \le 1 \tag{14}$$

The first main result of this note is derived from (13) and (14).

**Theorem 1** *Suppose that $\psi^G > \psi^L$. Then, $c_t \le d_t$ can hold if and only if $\varepsilon > 1$.*

**Proof.** Denote by $L(\omega_t)$ the left-hand side of (14) as a function of $\omega_t$. Note that $L(\omega_t)$ is monotonically decreasing in $\omega_t > 0$ whether $\varepsilon > 1$ or $\varepsilon < 1$. First see the necessity for the first part of Theorem 1; it suffices to show that $\varepsilon < 1$ necessarily implies $c_t > d_t$ when $\psi^G > \psi^L$. Suppose $\varepsilon < 1$. $L(\omega_t) \to \infty$ as $\omega_t \to 0$ and $L(\omega_t) \to \psi^G/\psi^L > 1$ as $\omega_t \to \infty$. Thus, $L(\omega_t) > 1$ for all $\omega_t > 0$, meaning $c_t > d_t$. Next see the sufficiency. Suppose $\varepsilon > 1$. Since $L(\omega_t) = \psi^G/\psi^L > 1$ as $\omega_t = 0$ and since $L(\omega_t) \to 0$ as $\omega_t \to \infty$, there uniquely exists $\hat{\omega}$ such that $c_t \le d_t$ holds for $\omega_t \in [\hat{\omega}, \infty]$. ■

Theorem 1 identifies a fundamental factor that determines whether global innovation occurs through collaboration or through outsourcing. It is the relative cost of global innovation to local innovation, $\psi^G/\psi^L$. When $\psi^G/\psi^L > 1$, $c_t \le d_t$ cannot hold unless $\varepsilon > 1$; global innovation cannot take place through collaboration ($\varepsilon < 1$) but can do through offshoring ($\varepsilon > 1$). Only if $\psi^G/\psi^L < 1$, $c_t \le d_t$ may hold; global innovation can occur through collaboration.

**Proposition 1** *So long as global innovation is costly relative to local innovation, innovation can never take place through global collaboration. It takes place through global sourcing.*

Next, I examine the role of skilled labor endowments, $R^H$ and $R^F$. To ensure global innovation to occur through collaboration, by Theorem 1, it is necessary to assume

$$\varepsilon < 1 \text{ and } \psi^G/\psi^L < 1. \tag{15}$$

Just for the simplicity of description, $\varepsilon$ and $\psi^G/\psi^L$ may be normalized such as $\varepsilon = 0.5$, $\psi^G = 1/2$, and $\psi^L = 1$. These normalizations never affect qualitatively the result shown below at all.

Towards this end, I will derive and characterize the equilibrium dynamical system of the model.[8] Before proceeding, I should relate the cumulative knowledge stock, $A_t^i$, to the number of goods. Following the standard literature (e.g., Romer 1990), the knowledge stock at date $t$ is defined as a linear function of the number of innovations (goods) that have been made up to date $t$: $A_t^i = N_t$.[9]

To derive the equilibrium system, first, by (14), $c_t \leq d_t$ holds with equality only at $\omega_t = \omega^*$ such that

$$\omega^* = (3 - 2\sqrt{2})^{-1}. \tag{16}$$

This ensures that global innovation can take place through collaboration or outsourcing. Then, by applying the Shephard's lemma to (2), the factor demand functions in global innovation are derived:

$$h_t^H = \sqrt{c_t w_{R,t}^H / 2N_t} \; / w_{R,t}^H, \tag{17}$$

$$h_t^F = \sqrt{c_t w_{R,t}^F / 2N_t} \; / w_{R,t}^F. \tag{18}$$

Finally, denoting the fraction of global innovations through collaboration as $n_t^G = \frac{N_t^G}{N_t}$ and the growth rate of technologies as $g_t = \frac{\dot{N}_t}{N_t}$, I can derive the equilibrium dynamical system for $n_t^G$. Assume

$$\frac{R^H}{R^F} > \sqrt{2} - 1. \tag{19}$$

Then I have the following theorem.

**Theorem 2** *The dynamics of the fraction of global collaboration $n_t^G$ obeys the following differential equation:*

$$\frac{\dot{n}_t^G}{n_t^G} = (2 - \sqrt{2}) R^F \left(n_t^G\right)^{-1} - (3 - 2\sqrt{2}) R^F - R^H. \tag{20}$$

[8] The analysis below is sufficient for the purpose but does not cover all. Appendix A provides a thorough derivation and characterization of the equilibrium dynamics of the model.

[9] It may be important to make the model structure as simple as possible since this is the first attemt to model innovation through global collaboration, for the spillover functions may be simplified to be symmetric about country. Nevertheless, note that it can be shown that allowing for incomplete international knowledge spillover (with $A_t^H > A_t^F$) does not affect the results qualitatively.

*Under (19), there exists the unique, globally stable balanced growth path, on which $n_t^G$ is constant at $\hat{n}^G$ that satisfies*

$$\hat{n}^G = \frac{\sqrt{2}\ (\sqrt{2}-1)R^F}{R^H + (3-2\sqrt{2})R^F}. \tag{21}$$

**Proof.** Recall the normalizations: $\varepsilon = 0.5$, $\psi^G = 1/2$, and $\psi^L = 1$. By cancelling out $\dot{N}_t^G$ from (8) and (9),

$$\frac{\dot{N}_t^H}{N_t^H} = \frac{1}{1-n_t^G}\left(R^H - R^F(\sqrt{2}-1)\right), \tag{22}$$

in which the use of (16), (17), and (18) has been made. By incorporating (22) back to (8),

$$\frac{\dot{N}_t^G}{N_t^G} = R^F\sqrt{2}(\sqrt{2}-1)\frac{1}{n_t^G}, \tag{23}$$

noting $c_t = d_t$ with (3). Since $\frac{\dot{N}_t}{N_t} = n_t^G\frac{\dot{N}_t^G}{N_t^G} + \left(1-n_t^G\right)\frac{\dot{N}_t^H}{N_t^H}$ from $N_t = N_t^G + N_t^H$, (22) and (23) imply

$$\frac{\dot{N}_t}{N_t} = R^F(\sqrt{2}-1)^2 + R^H, \tag{24}$$

noting (16). By (22) and (24), (20) can be derived. Since $\dot{n}_t^G = 0$ on a balanced growth path, (20) implies (21), noting (19). ■

This theorem demonstrates that the balanced growth level of the fraction of global innovation through collaboration $\hat{n}^G$ is determined by a key economic parameter: the skilled labor endowment of the foreign country relative to the home country, $R^F/R^H$. As (21) shows, an increase in the relative skilled-labor endowment of the foreign country, $R^F/R^H$, leads to an expansion of the fraction of global innovation through collaboration, $\hat{n}^G$. This implies that an increase in the home country's skilled labor, $R^H$, discourages global innovation through collaboration, a decrease in $\hat{n}^G$.

This negative effect of $R^H$ on $\hat{n}^G$ may seem to be odd but can be interpreted as a version of the Rybczynski effect in international economics. To understand how, interpret the innovation market of the present model as a two-good, two-factor market. The two goods are global and local innovations. The two factors are foreign skilled labor and home skilled labor, represented by $R^F$ and $R^H$. Note that local innovation uses home skilled labor $R^H$ intensively while global innovation uses foreign skilled labor $R^F$ intensively relative to local innovation. Applying the Rybczynski effect to the current model, an increase in the endowment of one factor (say home labor $R^H$), leads to a more than proportional expansion of the output in the sector which uses that factor, $R^H$, intensively (i.e., local innovation), and an absolute decline of the output of the other good (i.e., global innovation).

**Proposition 2** *Innovation through global collaboration, relative to local innovation, increases with the endowment of foreign skilled labor and decreases with the endowment of home skilled labor in the steady state equilibrium.*

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**Appendix A:**

In what follows, I will complete the characterization of dynamic equilibrium for the model without the assumption of (15).

Start with the consumers utility maximization. As is well known , after normalizing the instantaneous aggregate spending to 1 at each date ($E_t = \int_0^{N_t} p_t(j)\, x_t(j)\, dj = 1$), the dynamic optimization with (6) requires the equality of the discount rate and interest rate: $\rho = r_t$. By (7), the instantaneous aggregate demand for good $j$ is given by

$$x_t(j) = p_t(j)^{-\sigma} / \int_0^{N_t} p_t(j)^{1-\sigma} dj. \tag{25}$$

This implies that the price elasticity of demand for any good $j$ is constant at $\sigma > 1$.

Turn to the manufacturing firm's activity. Given (25) with $\sigma > 1$, the manufacturing firm solves (5) and sets the monopolistic price. Denote as $p_t^i$ the price that the firm located in Country $i$. Then I have

$$p_t(j) = \begin{cases} p_t^H = \frac{\sigma w_t^H}{\sigma-1} & \text{for } j \in \Lambda_t(H) \\ p_t^F = \frac{\sigma z w_t^F}{\sigma-1} & \text{for } j \in \Lambda_t(F) \end{cases}. \tag{26}$$

Denote as $\theta_t$ the fraction of goods that belong to $\Lambda_t(F)$, which is endogenously determined later. Then, $\theta_t N_t$ and $(1-\theta_t) N_t$ denote the numbers of goods manufactured in Country $F$ and Country $H$, respectively. From (5), (25), and (26), the demand function $x_t(j)$ the profit function can be written as

$$x_t(j) = \begin{cases} x_t^H = \frac{\sigma-1}{\sigma} \frac{\left(w_t^H\right)^{-\sigma}}{\theta_t N_t \left(z w_t^F\right)^{1-\sigma} + (1-\theta_t) N_t \left(w_t^H\right)^{1-\sigma}} & \text{for } j \in \Lambda_t(H) \\ x_t^F = \frac{\sigma-1}{\sigma} \frac{\left(z w_t^F\right)^{-\sigma}}{\theta_t N_t \left(z w_t^F\right)^{1-\sigma} + (1-\theta_t) N_t \left(w_t^H\right)^{1-\sigma}} & \text{for } j \in \Lambda_t(F) \end{cases} \tag{27}$$

and

$$\pi_t(j) = \begin{cases} \pi_t^H = \frac{w_t^H x_t^H}{\sigma-1} & \text{for } j \in \Lambda_t(H) \\ \pi_t^F = \frac{z w_t^F x_t^F}{\sigma-1} & \text{for } j \in \Lambda_t(F) \end{cases}. \tag{28}$$

Next, I demonstrate that those demands and profits can be rewritten as simple expressions in equilibrium by noting the equilibrium conditions (8), (9), and (10). The non-arbitrage condition for location (10), with (4), requires the profits in different countries $\pi_t^H$ and $\pi_t^F$ to be equated;

$\pi_t^H = \pi_t^F = \pi_t$ holds at each date $t$. From (28), this implies $w_t^H = zw_t^F$. By substituting $w_t^H = zw_t^F$ into (28), the profit function has the following simple expression:

$$\pi_t = \frac{1}{\sigma N_t}. \tag{29}$$

Next by substituting $w_t^H = zw_t^F$ to (27), $x_t^H = (\sigma - 1)/\sigma N_t w_t^H$ and $x_t^F = (\sigma - 1)/\sigma N_t z w_t^F$ hold. Incorporating these expressions into the market clearing conditions for unskilled labor in (8) and (9) can show that the wage rates for unskilled labor and the fraction of goods manufactured in the foreign country are independent of time:

$$w_t^H = \frac{z(\sigma - 1)}{\sigma(zL^H + L^F)} \text{ and } w_t^F = \frac{\sigma - 1}{\sigma(zL^H + L^F)}; \tag{30}$$

$$\theta_t = \frac{L^F}{zL^H + L^F}. \tag{31}$$

Also, by (30), good $j$'s amount produced, $x_t(j) = x_t^H$ or $x_t^F$, does not depend on its production location.

$$x_t^H = x_t^F = \frac{zL^H + L^F}{zN_t} = x_t. \tag{32}$$

By (4), (10), (11), and (29), the usual Bellman equation is derive.

$$r_t V_t = \frac{1}{\sigma N_t} + \dot{V}_t. \tag{33}$$

Departing the normalization of parameters, $\varepsilon$, $\psi^G$, and $\psi^L$, the growth rate can be written as

$$\frac{\dot{N}_t}{N_t} = \frac{R^H}{\psi^L} + R^F\left(\left(\psi^G\right)^{-(1-\varepsilon)} - \left(\psi^L\right)^{-(1-\varepsilon)}\right)^{\frac{1}{1-\varepsilon}} = g. \tag{34}$$

(The derivation of (34) is given in Appendix B.) Define $v_t = N_t V_t$. Noting $r_t = \rho$, (33) and (34) imply

$$\dot{v}_t = (g + \rho)\, v_t - 1/\sigma. \tag{35}$$

Note that the dynamic equilibrium of the model is characterized by two autonomous differential equations: (20) and (35).

A general solution to (35) can be written as

$$v_t = \left( v_0 - \frac{1}{\sigma\left(g+\rho\right)} \right) e^{(g+\rho)t} + \frac{1}{\sigma\left(g+\rho\right)}. \tag{36}$$

The usual transversality condition, coming from the consumer's dynamic optimization, requires that $v_t$ is bounded. Thus, from (36), the particular solution to (35) can be given by

$$v_0 = \frac{1}{\sigma\left(g+\rho\right)} \tag{37}$$

and

$$v_t = \frac{1}{\sigma\left(g+\rho\right)} = v^* \tag{38}$$

for all $t > 0$.

**Appendix B:**

The proof for (34) will be provided.

By (14),

$$w_{R,t}^{H} = \left(\left(\psi^{G}/\psi^{L}\right)^{-(1-\varepsilon)} - 1\right)^{-\frac{1}{1-\varepsilon}} w_{R,t}^{F}. \tag{39}$$

The wages of skilled labor can be determined using (12) and (39).

$$w_{R,t}^{H} = v^{*} = w_{R}^{H} \text{ and } w_{R,t}^{F} = \left(\left(\psi^{G}/\psi^{L}\right)^{-(1-\varepsilon)} - 1\right)^{\frac{1}{1-\varepsilon}} v^{*} = w_{R}^{F} \tag{40}$$

for all $t \geq 0$.

By applying Shephard's lemma to (1), the factor demands for skilled labor from global innovation firms are:

$$h_{t}^{H} = \left(\psi^{G}\right)^{1-\varepsilon} (w_{R,t}^{H}/N_{t})^{1-\varepsilon} (c_{t})^{\varepsilon} / w_{R,t}^{H} \tag{41}$$

and

$$h_{t}^{F} = \left(\psi^{G}\right)^{1-\varepsilon} (w_{R,t}^{F}/N_{t})^{1-\varepsilon} (c_{t})^{\varepsilon} / w_{R,t}^{F}, \tag{42}$$

noting $A_{t}^{i} = N_{t}$. From (8) and (9), these two imply

$$\frac{\dot{N}_{t}^{H}}{N_{t}^{H}} = \frac{1}{\psi^{L}\left(1 - n_{t}^{G}\right)} \left(R^{H} - R^{F}\left(\left(\psi^{G}/\psi^{L}\right)^{-(1-\varepsilon)} - 1\right)^{\frac{\varepsilon}{1-\varepsilon}}\right) \tag{43}$$

and

$$\frac{\dot{N}_{t}^{G}}{N_{t}^{G}} = \frac{R^{F}}{n_{t}^{G}\left(\psi^{G}\right)^{1-\varepsilon}\left(\psi^{L}\right)^{\varepsilon}} \left(\left(\psi^{G}/\psi^{L}\right)^{-(1-\varepsilon)} - 1\right)^{\frac{\varepsilon}{1-\varepsilon}}. \tag{44}$$

Then,

$$\frac{\dot{N}_{t}}{N_{t}} = \frac{R^{H}}{\psi^{L}} + \frac{R^{F}}{\psi^{L}} \left(\left(\frac{\psi^{G}}{\psi^{L}}\right)^{-(1-\varepsilon)} - 1\right)^{\frac{1}{1-\varepsilon}}. \tag{45}$$

For the general case, the assumption for Theorem 1, (19), should be replaced by

$$\frac{R^{H}}{R^{F}} > \left(\left(\psi^{G}/\psi^{L}\right)^{-(1-\varepsilon)} - 1\right)^{\frac{\varepsilon}{1-\varepsilon}}. \tag{46}$$