

# An Analysis of Monetary Policy in a Monetary Search Model with Non-unitary Discounting

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## Abstract

Based on findings in the behavioral economics literature, I incorporate non-unitary discounting into a monetary search model to study optimal monetary policy. I apply non-unitary discounting, that is, discount rates that are different across goods. With this extension to the model, I find that there are cases where optimal monetary policy deviates from the Friedman rule.

*Keywords:* Non-unitary discounting; Search; Friedman rule

*JEL classification:* E52, E70

## 1 Introduction

Recent research on behavioral economics find that subjective discount rates differ between goods (hereafter, following Hori and Futagami (2019), the phenomenon is denoted by “non-unitary discounting”). For example, based on an experiment conducted in Uganda, Ubfal (2016) reports the existence of non-unitary discounting.

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The problem of non-unitary discounting is that it causes time-inconsistent preference. For instance, suppose that an individual lives infinitely and has the following preference in period  $t$ :

$$U_t = \sum_{i=0}^{\infty} \beta_c^i u(c_{t+i}) + \beta_l^i v(l_{t+i}),$$

where  $c$  is consumption,  $l$  is leisure, and  $u(\cdot)$  and  $v(\cdot)$  are the utility functions of consumption and leisure. The difference between this and the standard models is that the discount factors of consumption and leisure are different. This means that at  $t = 0$ , the individual has the following preference:

$$U_0 = u(c_0) + v(l_0) + \beta_c u(c_1) + \beta_l v(l_1) + \beta_c^2 u(c_2) + \beta_l^2 v(l_2) + \cdots, \quad (1)$$

and at  $t = 1$ , the individual has a preference of

$$U_1 = u(c_1) + v(l_1) + \beta_c u(c_2) + \beta_l v(l_2) + \beta_c^2 u(c_3) + \beta_l^2 v(l_3) + \cdots. \quad (2)$$

I focus on the marginal rate of substitution between consumption and leisure in period 1. From (1), the individual's marginal rate of substitution in period 0 is given by  $[\beta_c u'(c_1)]/[\beta_l v'(l_1)]$ . On the other hand, from (2), in period 1, it is given by  $u'(c_1)/v'(l_1)$ . Because the marginal rate of substitutions between consumption and leisure in  $t = 0$  and  $t = 1$  are different, the individual's preference changes from  $t = 0$  to  $t = 1$ . Therefore, the preference of non-unitary discounting has time-inconsistency.

Because non-unitary discounting has time-inconsistent preference, an individual's intertemporal decision making may differ from the standard model without non-unitary discounting. This is shown by Hori and Futagami (2019) and Ohdoi et al. (2015), which study the general equilibrium model without money. Moreover, these studies show that the resource allocation is both different from the standard model and inefficient. I think that non-unitary discounting affects not only the real economy but also the monetary economy. If the individual's preference has time-inconsistency, the decision making around consumption is affected. Because money is held for the purpose of consumption, non-unitary discounting may also affect demand for money. Therefore, in this paper, I investigate the effect of non-unitary discounting on the monetary economy. As mentioned before, the resource allocation of an economy with non-unitary discounting is inefficient. This means that monetary policy may improve welfare. Therefore, the

aim of this paper is to discuss optimal monetary policy.

To achieve the above aims, I introduce non-unitary discounting to a monetary search model provided by Lagos and Wright (2005). In the Lagos-Wright model, each period is divided into two subperiods. In the first subperiod, a decentralized market is open and individuals must search for a partner with which to trade. Moreover, money is the intermediary of trade in the market. In the second subperiod, a centralized market is open and perfectly competitive. In each market, individuals obtain utility from consumption and disutility from supplying goods (labor). In my model, I assume that the discount rates of consumption and supplying the goods in each subperiod are different.

Through this extension, I show that the Friedman (1969) rule may be not optimal. This tends to occur in the following two cases. The first case is when the discount rate of the disutility from producing goods in the decentralized market is low. In this case, the sum of the discounted disutility from producing is emphasized more. Therefore, welfare is improved by decreasing the production of goods. Consumption in the decentralized market can be decreased by raising the inflation rate because an increase in the inflation rate raises the cost of holding money to pay for expenditures in the decentralized market. Therefore, welfare is improved by increasing inflation and thus decreasing consumption. The second case is when the discount rate for the utility of the consumption of goods traded in the decentralized market is sufficiently low. The demand for money is higher than in the unitary discounting case because the individuals want to consume more the next time they experience the first subperiod because of the low discount rate of goods. In this case, the consumption of goods in the decentralized market is higher than the optimal one. As in the first case, the high inflation rate decreases the money demand and consumption in a decentralized market, and it improves welfare. The Friedman rule is that the inflation rate should be minimized to reduce or eliminate loss from holding money. In both of the above cases, higher inflation improves welfare, leading to the conclusion that the Friedman rule is not optimal.

There are several studies that use experiments and interviews to show that the discount rate differs between goods. Odum and Rainaud (2003) study the difference in the discount rates of hypothetical money, food, and alcohol. Additionally, Odum et al. (2006) investigate this difference in hypothetical money and food. Attema et al. (2018) study the difference in the discount rates of health and money. There are several theoretical studies analyzing a monetary economy by employing an element of behavioral economics. Hori and Futagami (2021) study the

monetary economy with non-unitary discounting. However, they assume that households face a cash-in-advance constraint. My model has a finer micro-foundation for money demand than their model. Hiraguchi (2018) incorporates temptation preference into the Lagos-Wright model whereby the agents have a desire to spend all their money, and they experience disutility from suppressing that desire. The author shows that the Friedman rule may be not optimal. Although my model is similar to his model, the behavioral economic assumption is different. Moreover, the following studies analyze an economy with hyperbolic discounting: Gong and Zhu (2009), Boulware et al. (2013), Graham and Snower (2008, 2013), and Maeda (2018, 2023). There are also many theoretical studies that discuss the optimal monetary policy by using a search model, including Aruoba and Chugh (2010), Bhattacharya et al. (2005), Berentsen, Camera and Waller (2007), Berentsen, Rocheteau and Shi (2007), Faig and Jerez (2007), He et al. (2008), Hiraguchi and Kobayashi (2014), Hu et al. (2009), Jeong (2015), Lagos and Rocheteau (2005), Li (1995), Shi (1997, 2001), and Williamson (2006). These studies basically focus on market structures and bargaining but do not focus on the discounting rate.

The remainder of this paper is organized as follows: Section 2 provides the model. Section 3 addresses the individual’s optimization. Section 4 obtains the equilibrium. Section 5 presents an analysis of government policy. Section 6 concludes the paper.

## 2 The model

In this section, I explain the environment. Time is discrete and runs from  $t = 0$  to  $\infty$ . There exists a continuum of infinitely lived agents with a unit measure. Each period is divided into two subperiods. The first subperiod is called “day” and the second is called “night.”

In the day subperiod, the individuals become either a “seller,” who produces goods with probability  $n \in (0, 1)$ , or a “buyer,” who consumes it with probability  $1 - n$ , before they enter a market<sup>1</sup>. The market is perfectly competitive<sup>2</sup>. Following previous studies, I call the market a decentralized market or DM. The buyers must use money to trade because it is assumed that buyers and sellers are anonymous.

In the night subperiod, the individuals determine consumption, the supply of labor, and the

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<sup>1</sup>In the original setting of the Lagos-Wright model, the individuals enter the market and search for their partner. Then, they find whether they become a seller or buyer. This point differs from my model. However, my model’s setting can obtain the same result as the Lagos-Wright model when the price bargaining power of the buyer is equal to one. Because my model’s setting is simpler than the Lagos-Wright model, I use the simpler one. I show that my model’s setting can obtain the same result as the Lagos-Wright model in Appendix C.

<sup>2</sup>There are previous studies in which the seller or buyer is determined exogenously, and the market is perfectly competitive. For example, Berentsen, Camera, and Waller (2007).

amount of money carried over to the next period at the same time. In other words, the market is a standard centralized perfectly competitive market. I call the market a centralized market or CM. One unit of a good in this market is produced by inputting one unit of labor.

Money is divisible and storable but intrinsically useless.  $M_t$  denotes the amount of money issued before period  $t$ , and the growth rate of money is  $\gamma - 1$ . Therefore,  $M_{t+1}/M_t = \gamma$ . Money is issued in the night subperiod, and all money is transferred to individuals equally. When we define  $T$  as the transfer of money to individuals, we obtain  $T_t = (\gamma - 1)M_t$ .

In the rest of this section, I explain the assumption about the utility of the individual. The individual obtains utility from consumption and disutility from production and labor supply in each subperiod. As in the Lagos-Wright model, we assume that the utility function is additively separable and quasilinear; that is

$$\mathcal{U}(q_t^b, q_t^s, x_t, h_t) = u(q_t^b) - c(q_t^s) + U(x_t) - h_t, \quad (3)$$

where  $u(q)$  is the utility from consuming  $q$  units of the goods and  $c(q)$  is the cost of producing  $q$  units of the goods in the day subperiod.  $U(x)$  is the utility from consuming  $x$  units of the goods, and  $h$  is the disutility of supplying  $h$  units of labor in the night subperiod. The functions  $u$ ,  $c$ , and  $U$  are twice continuously differentiable and satisfy  $u(0) = c(0) = 0$ ,  $u' > 0$ ,  $c' > 0$ ,  $U' > 0$ ,  $u'' < 0$ ,  $c'' \geq 0$ ,  $U'' < 0$ ,  $u'(0) = U'(0) = \infty$ , and  $u'(\infty) = U'(\infty) = 0$ .  $h$  is positive and has upper bound  $\bar{h}$ . The key assumption of this paper is that the discount rates for the (dis)utility of consumption and production are different. Therefore, I consider the following lifetime utility in period  $t$ :

$$Z_t = \sum_{i=0}^{\infty} \left[ \beta_{db}^i u(q_{t+i}^b) - \beta_{ds}^i c(q_{t+i}^s) + \beta_{cb}^i U(x_{t+i}) - \beta_{cs}^i h_{t+i} \right], \quad (4)$$

where  $\beta_{bd}$  denotes the discount factor of  $u(q^b)$ ,  $\beta_{ds}$  denotes the discount factor of  $c(q^s)$ ,  $\beta_{cb}$  denotes the discount factor of  $U(x)$ , and  $\beta_{cs}$  denotes the discount factor of  $h$ . As mentioned in the introduction, non-unitary discounting causes time inconsistency. Therefore, it is important to determine whether the individuals are sophisticated or naive. In this paper, we assume that all individuals are sophisticated.

### 3 Individual's optimization

As mentioned in the previous section, there are two states associated with an agent: a seller and a buyer. In this section, we seek the optimal behavior in each state. After that, we seek the amount of money brought to the next period. In terms of notations, I omit the subscript  $t$ , which represents added variables, except for the case in which it is needed. Moreover, I add the variable in the next period to  $+1$ , for example,  $z_{+1}$ .

#### 3.1 Seller's problem

First, we seek the seller's optimal behavior. The individual who becomes a seller in period  $t$  can obtain money and experience disutility,  $c(q^s)$ , in the day subperiod. The seller also experiences net utility,  $U(x) - h$ , in the night subperiod. Therefore, the seller's value function, which she maximized, is given by

$$V_0^s(m) = \max_{q^s, x, h, m_{+1}} [-c(q^s) + U(x) - h + V(m_{+1})], \quad (5)$$

where

$$V(m_{+1}) \equiv \beta_{db}V_{db}(m_{+1}) - \beta_{ds}V_{ds}(m_{+1}) + \beta_{cb}V_{cb}(m_{+1}) - \beta_{cs}V_{cs}(m_{+1}). \quad (6)$$

$V(m)$  is the value that is obtained from future consumption and production. Each element,  $V_{db}(m)$ ,  $V_{ds}(m)$ ,  $V_{cb}(m)$ , and  $V_{cs}(m)$ , denotes the sum of the expected discounted present value of consuming and producing goods in the future day subperiod, and consuming and supplying labor in the future night subperiod, respectively. These are given by

$$V_{db}(m_t) = E \sum_{j=0}^{\infty} \beta_{db}^j u(q_{t+j}^b), \quad (7)$$

$$V_{ds}(m_t) = E \sum_{j=0}^{\infty} \beta_{ds}^j c(q_{t+j}^s), \quad (8)$$

$$V_{cb}(m_t) = E \sum_{i=0}^{\infty} \beta_{cb}^i U(x_{t+i}), \quad (9)$$

$$V_{cs}(m_t) = E \sum_{i=0}^{\infty} \beta_{cs}^i h_{t+i}, \quad (10)$$

where  $E$  is the expectation operator. The seller's budget constraint in the night subperiod is given by

$$h = x + \phi m_{+1} - \phi p q^s - \phi(m + T) \quad (11)$$

where  $\phi$  is the real price of money and  $p$  is the nominal price of the goods in the day subperiod. Substituting (11) into (5), we obtain

$$V_0^s(m) = \max_{q^s} [\phi p q^s - c(q^s)] + \max_x [U(x) - x] + \phi(m + T) + \max_{m_{+1}} [-\phi m_{+1} + V(m_{+1})]. \quad (12)$$

Because we assume that the DM is perfectly competitive, the seller treats  $\phi p$  as given. Therefore, the first order condition of  $q^s$  is given by

$$\phi p = c'(q^s). \quad (13)$$

From the second term of (12), the first order condition of  $x$  is given by

$$U'(x) = 1. \quad (14)$$

Let  $x^*$  denote the value of satisfying (14). Moreover, we assume that  $U(x^*) > x^*$  is satisfied for  $x^*$  to have a positive value. From the last term of (12), we find that  $m_{+1}$  does not depend on  $m$  or on whether the individual becomes the seller or buyer in the day subperiod. This is a property of the Lagos-Wright model. Because of this property, we do not have to consider each individual's state. Although we should seek the first order condition of  $m_{+1}$ , we will obtain it in subsection 3.3 because  $V(m_{+1})$  depends not only on the seller's behavior but on the buyer's, which is discussed in the next subsection.

### 3.2 Buyer's problem

In this subsection, we seek the buyer's optimal behavior. Because buyers and sellers are anonymous, buyers need to hold money before they consume. Therefore, buyers face the following constraint:

$$p q^b \leq m. \quad (15)$$

The buyer's budget constraint in the night subperiod is given by

$$h = x + \phi m_{+1} + \phi p q^b - \phi(m + T). \quad (16)$$

Calculating as in the seller's problem, we obtain the value function, which the buyer maximizes, as follows:

$$V_0^b(m) = \max_{pq^b \leq m} [u(q^b) - \phi p q^b] + \max_x [U(x) - x] + \phi(m + T) + \max_{m_{+1}} [-\phi m_{+1} + V(m_{+1})]. \quad (17)$$

From this equation, we find that the optimal conditions of  $x$  and  $m_{+1}$  are the same as those of the seller. The value of  $q^b$  is determined by whether (15) is binding. If (15) is not binding,  $p q^b < m$ , then the individual determines  $q^b$  to satisfy  $u'(q^b) = \phi p$ . If (15) is binding,  $q^b = m/p$ . Furthermore, from the first term of (17), the buyer has no incentive to increase the consumption in the DM to satisfy  $u'(q^b) < \phi p$  in the binding case because we assume that  $u''(q^b) < 0$ . Therefore, the following condition is always satisfied:

$$u'(q^b) \geq \phi p. \quad (18)$$

### 3.3 Money holdings

To determine the amount of money brought to the next period, we must obtain the future value,  $V(m_{+1})$ . From the definitions of  $V(m_{+1})$ , (6), and (7) to (10), we can express the future value as follows:

$$\begin{aligned} V(m_{+1}) = & \beta_{db}[(1-n)u(q_{+1}^b) + \beta_{db}V_{db}(m_{+2})] - \beta_{ds}[nc(q_{+1}^s) + \beta_{ds}V_{ds}(m_{+1})] \\ & + \beta_{cb}[(1-n)U(x_{+1}^b) + nU(x_{+1}^s) + \beta_{cb}V_{cb}(m_{+2})] \\ & - \beta_{cs}[(1-n)h_{+1}^b + nh_{+1}^s + \beta_{cs}V_{cs}(m_{+2})], \end{aligned} \quad (19)$$

where  $m_{+2}$  denote money holdings after two periods,  $x^{b(s)}$  is the consumption, and  $h^{b(s)}$  is labor supply in the CM when the individual becomes the buyer (seller). The variables of (19) are the future quantity. Therefore, it is important to understand how the individual predicts her future behavior. In this paper, we assume that the individual is sophisticated. That individual knows that she cannot control her future behavior. In other words, her future self maximizes



(12) and (17) just as her current self does.<sup>3</sup> From subsections 3.1 and 3.2 which describe current behavior,  $q^s$ ,  $x$  and  $m_{+1}$  do not depend on  $m$ . Since this implies that  $q_{+1}^s$ ,  $x_{+1}$ , and  $m_{+2}$  do not also depend on  $m_{+1}$ , (19) is rewritten as follows:

$$V(m_{+1}) = (1 - n)v(m_{+1}) + \beta_{cs}\phi_{+1}m_{+1} + \text{constant term}, \quad (20)$$

where  $v(m_{+1}) \equiv \beta_{db}u(q_{+1}^b) - \beta_{cs}\phi_{+1}p_{+1}q_{+1}^b$  is the benefit for the current individual when the individual becomes the buyer in the next period, *constant term* is a term that does not depend on  $m_{+1}$ , and (20) has been substituted as the budget constraint of the night subperiod, (11) and (16). Substituting this equation into the last term of (12) and (17), and omitting the terms that do not depend on  $m_{+1}$ , we obtain

$$\max_{m_{+1}} [-\phi m_{+1} + (1 - n)v(m_{+1}) + \beta_{cs}\phi_{+1}m_{+1}]. \quad (21)$$

If (15) is binding, we obtain the first order condition as follows:

$$\frac{\phi}{\phi_{+1}} = (1 - n)\beta_{db}\frac{u'(q_{+1}^b)}{\phi_{+1}p_{+1}} + n\beta_{cs}. \quad (22)$$

We can also consider the cases where (15) is not binding. However, we can show that these cases are not equilibrium, as in Appendix A. Therefore, in subsequent sections, we only consider the binding case where (22) is satisfied.

## 4 Equilibrium

In this section, we discuss the equilibrium in this model. It is defined as follows.

**Definition 1.** *Equilibria consist of the quantities,  $q^s, q^b, x, h$ , and  $m_{+1}$ , and the prices,  $\phi$  and  $p$ , which satisfy*

1. *given the policy variable,  $\gamma$ , the individual behaves,*
2. *the current behavior satisfies (13), (14), and (22),*
3. *(15) is binding,*

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<sup>3</sup>There is another solution in which the individual can control her future behavior. In such a case, she is considered naive. The solution in this case may be different. However, this paper focuses on a sophisticated agent.

4. the current individual predicts future behavior in the same way as current behavior,

5. and the following market clearing conditions are satisfied:

(a) The DM goods

$$(1 - n)q^b = nq^s, \quad (23)$$

(b) The goods in the CM

$$x^* = (1 - n)h^b + nh^s, \quad (24)$$

(c) Money

$$m = M. \quad (25)$$

Following this definition, the inflation rate in equilibria is given by

$$\frac{\phi}{\phi_{+1}} = (1 - n)\beta_{db} \frac{u'(q_{+1})}{c'(\frac{1-n}{n}q_{+1})} + n\beta_{cs}. \quad (26)$$

where  $q$  is the amount of the consumption in the DM in equilibria. Because (15) is binding and  $m = M$ ,  $pq = M$ . Then, we can rewrite (26) as follows:

$$\frac{\phi pq}{\phi_{+1} p_{+1} q_{+1}} \frac{M_{+1}}{M} = (1 - n)\beta_{db} \frac{u'(q_{+1})}{c'(\frac{1-n}{n}q_{+1})} + n\beta_{cs}. \quad (27)$$

Because  $M_{+1}/M = \gamma$  and the seller's optimal condition, (13), is satisfied in the equilibria, from (27), we obtain the dynamics of  $q$  as follows:

$$c' \left( \frac{1-n}{n} q \right) q = \frac{c'(\frac{1-n}{n}q_{+1}) q_{+1}}{\gamma} \left[ (1 - n)\beta_{db} \frac{u'(q_{+1})}{c'(\frac{1-n}{n}q_{+1})} + n\beta_{cs} \right], \quad (28)$$

From this equation, we can draw the phase diagram shown in Figure 1.<sup>4</sup> From the figure, we find that there are two steady states. One is  $q = 0$ . The other is  $q = \bar{q} > 0$ . We also find that there are two types of equilibria in this model. One is that the economy approaches  $q = 0$ . The other is that the economy initially jumps to  $q = \bar{q}$ . The existence of two equilibria is also seen

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<sup>4</sup>See Appendix B for an explanation of how the figure is drawn.

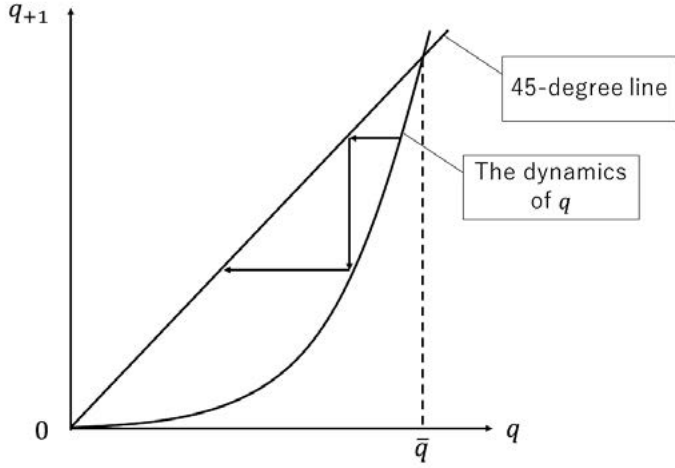


Figure 1: Phase diagram of  $q$

in the Lagos-Wright model. This is a property of the model introducing fiat money. This paper focuses on the latter equilibrium because it is realistic.

**Assumption 1.** *The equilibrium in which the economy initially jumps to  $q = \bar{q}$  is chosen.*

If we accept this assumption and combine (26) and (28), we obtain

$$\frac{\phi}{\phi_{+1}} = \gamma = (1 - n)\beta_{db} \frac{u'(\bar{q})}{c'(\frac{1-n}{n}\bar{q})} + n\beta_{cs}. \quad (29)$$

However,  $\bar{q}$  and  $\gamma$  have restrictions. Substituting (11) into (18), we obtain

$$\frac{u'(\bar{q})}{c'(\frac{1-n}{n}\bar{q})} > 1. \quad (30)$$

Notice that (30) does not include the equality because the case where  $\frac{u'(q)}{c'(\frac{1-n}{n}q)} = 1$  is not an equilibrium.<sup>5</sup> From (21), we obtain the restriction of the inflation rate. Because the individual does not need to have more money than her desired expenditure, the marginal benefit when she becomes a buyer in the next period,  $v'(m)$ , is not negative. Therefore, if  $-\phi + \beta_{cs}\phi_{+1} > 0$ , from (21), the marginal benefit of holding money is positive. This causes infinite money demand. To

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<sup>5</sup>This is shown in Appendix A.

hold the money market clearing condition, (25), the following relationship must hold:

$$\frac{\phi}{\phi_{+1}} \geq \beta_{cs}. \quad (31)$$

From (30) and (31), we obtain the restriction of  $\gamma$ :

$$\gamma \geq \underline{\gamma} \quad \text{where} \quad \underline{\gamma} = \max[\beta_{cs}, (1-n)\beta_{db} + n\beta_{cs}]. \quad (32)$$

In some cases,  $\gamma = \underline{\gamma}$  cannot hold. Therefore, (32) is a necessary condition for the equilibrium to exist.<sup>6</sup>

## 5 Policy analysis

In this subsection, we discuss the optimal monetary policy. First, I must define a welfare function. It is difficult to define this function because preferences change as time goes by when there is time-inconsistency. In this paper, the welfare function is given by the sum of the expected values at the beginning of a period or before the individuals find out whether they will become the seller or the buyer; that is,

$$W(m) \equiv nV_0^s(m) + (1-n)V_0^b(m). \quad (33)$$

I think that this welfare function is reasonable. Since the individuals in all periods face the problem, (12) and (17), and they can be both a seller and a buyer, defining (28) as a welfare function that the government maximizes is not wrong. Calculating each value, (7) to (10), in the equilibrium and substituting them into (33), the value of (33) in the equilibrium is given by

$$W = (1-n) \frac{u(\bar{q})}{1-\beta_{db}} - n \frac{c\left(\frac{1-n}{n}\bar{q}\right)}{1-\beta_{ds}} + \frac{U(x^*)}{1-\beta_{cb}} - \frac{x^*}{1-\beta_{cs}}. \quad (34)$$

From this equation, we obtain the following lemma about welfare.

**Lemma 1.**  $q^{**}$  maximizes the welfare function, (34), where  $q^{**}$  satisfies

$$\frac{u'(q^{**})}{c'\left(\frac{1-n}{n}q^{**}\right)} = \frac{1-\beta_{db}}{1-\beta_{ds}}. \quad (35)$$

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<sup>6</sup>We can also obtain the dynamics of the nominal price of the goods in the DM. Using (13), (29), and  $q = \bar{q}$  for all periods, it is given by  $p_{+1}/p = \gamma$ .

*Proof.* The first order condition of (34) is given by (35). Since  $u''(q^b) < 0$  and  $c''(q^s) \geq 0$ , the sufficient condition is satisfied. Therefore,  $q^{**}$  maximizes the welfare function.  $\square$

This lemma implies that if the discount factor of consumption in the DM,  $\beta_{db}$ , is high (low) –or that of the production in the DM is low (high),  $\beta_{ds}$ – the welfare maximizing level of consumption in the DM,  $q^{**}$ , is high (low). This is intuitive. A high (low)  $\beta_{db}$  denotes a high (low) evaluation of future consumption, and a low (high)  $\beta_{ds}$  denotes a low (high) evaluation of future disutility from production. This means that the welfare function, (34), is increased by an increase (decrease) in consumption and production. Therefore, if  $\beta_{db}$  is high (low) and  $\beta_{ds}$  is low (high),  $q^{**}$  is also high (low).

Next, we discuss the optimal policy. Since we have obtained the welfare maximizing level of  $q$  and the relationship between  $q$  and  $\gamma$ , (29), we easily obtain the optimal value of  $\gamma$  as follows.

**Proposition 1.** *The optimal policy is given by*

1.  $\gamma^* = \max \left[ (1-n)\beta_{db} \frac{1-\beta_{db}}{1-\beta_{ds}} + n\beta_{cs}, \beta_{cs} \right]$  if  $\beta_{db} < \beta_{ds}$ ,
2.  $\gamma^* \rightarrow \underline{\gamma}$  if  $\beta_{db} \geq \beta_{ds}$ .

*Proof.* In the first case, (35) is larger than one. From (30),  $q^{**}$  is feasible. Substituting (35) into (29), we obtain

$$\gamma = (1-n)\beta_{db} \frac{1-\beta_{db}}{1-\beta_{ds}} + n\beta_{cs}. \quad (36)$$

If this equation is larger than  $\beta_{cs}$ , it is the optimal policy because (32) is satisfied. Otherwise,  $\beta_{cs}$ , which is the lower limit of the inflation rate, is optimal.

In the second case, because (35) is smaller than one,  $q^{**}$  is not feasible. Moreover,  $q^{**} > q^*$ , where  $q^*$  satisfies  $u'(q^*)/c'(\frac{1-n}{n}q^*) = 1$ , because  $u''(q^b) < 0$  and  $c''(q^s) \geq 0$ . If so, the optimal policy is that  $q$  gets closer to  $q^*$ . Substituting  $u'(q^*)/c'(\frac{1-n}{n}q^*) = 1$  into (29), we obtain:

$$\gamma = (1-n)\beta_{db} + n\beta_{cs}. \quad (37)$$

The larger of (37) and  $\beta_{cs}$  is the optimal policy. This is the second case.  $\square$

We compare the optimal policy with the Friedman rule to evaluate the optimal policy. The Friedman rule is that the inflation rate should be minimized to minimize loss when holding

money. In this model, since the lower limit of the inflation rate is given by (32), we can define the Friedman rule as follows.

**Definition 2.** *The Friedman rule is a policy of minimizing the inflation rate; that is  $\gamma = (-)\underline{\gamma}$ .*

From this definition, we obtain the following proposition.

**Proposition 2.** *The Friedman rule is not optimal if  $\beta_{db} < \beta_{ds}$  and  $\frac{\beta_{db}}{\beta_{cs}} \frac{1-\beta_{db}}{1-\beta_{ds}} > 1$ .*

*Proof.* To show Proposition 2, we have to show that  $\gamma^* > \underline{\gamma}$ . In the case of this proposition, the optimal policy is  $\gamma^* = (1-n)\beta_{db} \frac{1-\beta_{db}}{1-\beta_{ds}} + n\beta_{cs} > \beta_{cs}$  because  $\frac{\beta_{db}}{\beta_{cs}} \frac{1-\beta_{db}}{1-\beta_{ds}} > 1$ . Hence, we show that  $\gamma^*$  is higher than  $(1-n)\beta_{db} + n\beta_{cs}$  to show that  $\gamma^* > \underline{\gamma}$ . This is true because

$$\gamma^* - (1-n)\beta_{db} + n\beta_{cs} = \beta_{db} \left( \frac{1-\beta_{db}}{1-\beta_{ds}} - 1 \right) > 0, \quad (38)$$

where  $\beta_{db} < \beta_{ds}$ . By the above, Proposition 2 is shown.

I also show that the Friedman rule is optimal in other cases. It is obvious that the Friedman rule is optimal if  $\beta_{db} \geq \beta_{ds}$  from Proposition 1. In the case where  $\frac{\beta_{db}}{\beta_{cs}} \frac{1-\beta_{db}}{1-\beta_{ds}} \leq 1$ ,  $\gamma^* = \beta_{cs}$  from Proposition 1. This means that  $\gamma^* \leq \underline{\gamma}$ . Therefore, the Friedman rule is optimal.  $\square$

This proposition implies that the Friedman rule tends not to be optimal in the following two cases. The first case is when the discount factor of production in the DM,  $\beta_{ds}$ , is high and that of consumption in the DM,  $\beta_{db}$ , is low. In this case, since the individual emphasizes the disutility from production more than the utility from consumption in the DM, the consumption maximizing welfare,  $q^{**}$ , is small. Consumption in the DM can be decreased by raising the inflation rate because an increase in the inflation rate raises the cost of holding money to pay the expenditure in the DM. Therefore, welfare is improved through an increase in inflation and thus a decrease in consumption. The second case is when the discount factor of consumption in the DM,  $\beta_{db}$ , is high and that of labor in the CM,  $\beta_{cs}$  is low.<sup>7</sup> In this case, the money demand is strong because the current individuals consume more in the next period. In some case, the individuals hold more money than the quantity needed to achieve optimal consumption. As mentioned before, an increase in inflation raises the cost of holding money. Therefore, welfare is improved by raising the inflation rate until the individual's money holding is optimal. In both

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<sup>7</sup>The small  $\beta_{cs}$  induces a high consumption demand like  $\beta_{db}$ . If the individual becomes the buyer, she has to increase the labor supply to obtain money for the future consumption (see (16)). The small  $\beta_{cs}$  means the future disutility from labor is small when becoming the buyer. That is, the cost of consumption is low when  $\beta_{cs}$  is small. Therefore, a small  $\beta_{cs}$  induces a high consumption demand.

of the above cases, a higher inflation improves welfare; thus, we can conclude that the Friedman rule is not optimal.

## 6 Concluding remarks

I developed a monetary search model that incorporates non-unitary discounting and analyzed the optimal monetary policy. Through the analysis, I showed that the Friedman rule may not be optimal. The first reason for this is that optimal consumption in the DM is lower than the standard model with unitary discounting. Because an increase in the inflation rate raises the cost of holding money, the consumption is decreased. Therefore, welfare is improved by raising the inflation rate. This case occurs when the discount rate (factor) of production is low (high) and that of consumption in the DM is high (low) because the individual emphasizes disutility from production more than utility from consumption in the DM. The second reason is that the demand for money is strong enough to increase future consumption. Then, the realized consumption is larger than the optimal one. Because a high inflation rate raises the cost of holding money, it is optimal. This case occurs when the discount rate (factor) of consumption in the DM is low (high) and that of labor in the CM is high (low). The Friedman rule is a policy of minimizing the inflation rate. Therefore, in both of the above cases, the Friedman rule may not be optimal. By the above discussion, I conclude that the difference in the discount rates significantly affects the optimality of the Friedman rule.

## Appendix

### A Proof of binding (15) in the equilibrium

Suppose that  $\beta_{cs}\phi_{+1} > \phi$ . Then, from (21), we find that the optimal behavior is  $m_{+1} = \infty$ . Therefore, this case is not the equilibrium and  $\beta_{cs}\phi_{+1} \leq \phi$  in the equilibrium. When  $\beta_{cs}\phi_{+1} < \phi$ , (15) is binding because the agents suffer losses if they hold more money than necessary. Therefore, we check if in the case where  $\beta_{cs}\phi_{+1} = \phi$ , there is no equilibrium or (15) binds. In this case,  $v'(m_{+1}) = 0$  is satisfied. This occurs in the following two cases. The first case is when  $\phi_{+1}p_{+1}q^* \leq m_{+1}$ , where  $q^{***}$  satisfies  $u'(q^{***}) = \phi p$ . In this case, the future individual does not increase consumption of the DM goods if  $m_{+1}$  increases. Therefore,  $v'(m_{+1}) = 0$ .

When we seek the left-hand limit of  $v'(m_{+1})$  at  $\phi_{+1}p_{+1}q^{***}$ , we get

$$\lim_{m_{+1} \rightarrow \phi_{+1}p_{+1}q^{***}-0} v'(m_{+1}) = -\phi_{+1}p_{+1} < 0.$$

Hence, the case where  $\phi_{+1}p_{+1}q^{***} = m_{+1}$  is not the equilibrium, and the case where  $\beta_{cs}\phi_{+1} < \phi$  is the only equilibrium.

The second case is when  $m_{+1} = \phi p q_{max}$ , where  $q_{max}$  denotes the maximizing value of  $v(m_{+1})$ . The individual does not have an incentive to increase her present holding of money because she does not want to consume more than  $q_{max}$  in the future. Therefore, (15) is binding in this case. By the above discussion, we show that (15) is binding in the equilibrium.

I also mention (30). Combining (13) and (23), in the equilibrium,  $\phi p = c'(\frac{1-n}{n}\bar{q})$ . Therefore,  $u'(q^*) = c'(\frac{1-n}{n}q^*)$  is not the equilibrium. This is why (30) does not include the equality.

## B The proof of drawing Figure 1

Totally differentiating (28), we obtain

$$\begin{aligned} \frac{dq_{+1}}{dq} = & \gamma \left[ c'' \left( \frac{1-n}{n}q \right) \frac{1-n}{n}q + c' \left( \frac{1-n}{n}q \right) \right] \\ & \times \left[ \left\{ c'' \left( \frac{1-n}{n}q_{+1} \right) \frac{1-n}{n}q_{+1} + c' \left( \frac{1-n}{n}q_{+1} \right) \right\} \left\{ (1-n)\beta_{db} \frac{u'(q_{+1})}{c'(\frac{1-n}{n}q_{+1})} + n\beta_{cs} \right\} \right. \\ & \left. + q_{+1}(1-n)\beta_{db} \frac{u''(q_{+1})c'(\frac{1-n}{n}q_{+1}) - u'(q_{+1})c''(\frac{1-n}{n}q_{+1})}{c'(\frac{1-n}{n}q_{+1})} \right]^{-1}. \end{aligned} \quad (B.1)$$

From (28), one of the steady states is  $q = q_{+1} = 0$ . When the above equation is evaluated near this steady state, we obtain

$$\frac{dq_{+1}}{dq} \Big|_{q=q_{+1}=0} = 0 \quad (B.2)$$

because  $u'(0) = \infty$ . In the other steady state, since (29) is satisfied,

$$\frac{dq_{+1}}{dq} \Big|_{q=q_{+1}=\bar{q}} = \left[ 1 + \bar{q}(1-n)\beta_{db} \frac{u''(\bar{q})c'(\frac{1-n}{n}\bar{q}) - u'(\bar{q})c''(\frac{1-n}{n}\bar{q})}{c'(\frac{1-n}{n}\bar{q}) \left[ c''(\frac{1-n}{n}\bar{q}) \frac{1-n}{n}\bar{q} + c'(\frac{1-n}{n}\bar{q}) \right] \gamma} \right]^{-1}. \quad (B.3)$$

Since  $u' > 0$ ,  $c' > 0$ ,  $u'' < 0$  and  $c'' \geq 0$ ,  $\frac{u''(\bar{q})c'(\frac{1-n}{n}\bar{q}) - u'(\bar{q})c''(\frac{1-n}{n}\bar{q})}{c'(\frac{1-n}{n}\bar{q}) \left[ c''(\frac{1-n}{n}\bar{q}) \frac{1-n}{n}\bar{q} + c'(\frac{1-n}{n}\bar{q}) \right] \gamma} < 0$ . Therefore, (B.3) is larger than 1. By the above discussion, we can draw Figure 1.



## C The case where the price and quantity of the goods in the DM are determined by bargaining

In this appendix, we change the assumptions of the DM from this paper as follows. Each individual produces and consumes a kind of good in the DM. In the DM, individual  $i$  searches for the person who consumes the goods produced or produces the goods consumed by individual  $i$ .  $\lambda \in (0, 1/2]$  denotes a probability of meeting a partner who can trade. There is no double coincidence of wants. The buyer and seller determine the quantity,  $q$ , and the price,  $p$ , of the DM good by Nash bargaining.

Because we do not change the other assumptions, the seller's payoff is given by (12) and the buyer's one is given by (17). Therefore, the bargaining problem is given by:

$$\max V_0^b(m)^\theta V_0^s(m)^{1-\theta} \quad (\text{C.1})$$

$$z \leq m, \quad (\text{C.2})$$

where  $z \equiv pq$  and  $\theta$  is the buyer's bargaining weight. Since the buyer's and seller's problem in the night subperiod does not change from subsection 3.1 and 3.2,  $x$  and  $m_{+1}$  do not depend on whether the individual becomes the buyer, the seller or other. Therefore, maximizing (C.1) is the same as maximizing the following equation:

$$(u(q) - \phi z)^\theta (-c(q) + \phi z)^{1-\theta}, \quad (\text{C.3})$$

If (C.2) is binding, the solution is given by

$$\phi z = g(q) \equiv \frac{\theta c(q)u'(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}. \quad (\text{C.4})$$

Notice that  $g'(q) > 0$  because there are assumptions of instant utility. If (C.2) is not binding,  $q = q^*$  because the optimal condition is  $u'(q^b) = c'(q^s)$ .

If  $\lambda < 1/2$ , there is a probability of not meeting the partner. Then, we can rewrite (19) as

follows:

$$\begin{aligned}
V(m_{+1}) = & \beta_{db}[\lambda u(q_{+1}^b) + \beta_{db}V_{db}(m_{+2})] - \beta_{ds}[\lambda c(q_{+1}^s) + \beta_{ds}V_{ds}(m_{+1})] \\
& + \beta_{cb}[U(x^*) + \beta_{cb}V_{cb}(m_{+2})] \\
& - \beta_{cs}[\lambda h_{+1}^b + \lambda h_{+1}^s + (1 - 2\lambda)h_{+1}^n + \beta_{cs}V_{cs}(m_{+2})],
\end{aligned} \tag{C.5}$$

where  $h^n$  is the labor supply when the individual cannot find her partner, and I have substituted the optimal consumption in the CM,  $x^*$ . Since

$$h^n = x + \phi m_{+1} - \phi(m + T),$$

we can rewrite (21) as follows:

$$\max_{m_{+1}} [-\phi m_{+1} + \lambda v(m_{+1}) + \beta_{cs}\phi_{+1}m_{+1}]. \tag{C.6}$$

Because the individual does not have any incentive to make her consumption larger than  $q^*$ , as in the case of the competitive DM,  $z_{+1} = m_{+1}$  in the equilibrium. Then, from (C.4),  $q'(m_{+1}) = \frac{\phi}{g'(q_{+1})}$ . Using this equation, we obtain (22) in the case of

$$\frac{\phi}{\phi_{+1}} = \lambda \beta_{db} \frac{u'(q_{+1})}{g'(q_{+1})} + (1 - \lambda) \beta_{cs}. \tag{C.7}$$

If  $\theta \rightarrow 1$ ,  $g(q) = c(q)$ . Then, (C.7) is the same as (26). In other words, if the price and quantity are determined by bargaining, we can obtain the same result in this paper.

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