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Fiscal policy and economic growth  
in the imperfect labor market

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### Abstract

Using a simple overlapping generations model with the growth engine of public capital by incorporating the union wage setting, we examine the effects of fiscal policies on unemployment and economic growth in the imperfect labor market. In particular, we demonstrate analytically that the growth-maximizing tax in the imperfect labor market is larger than that of the perfect labor market taking into account a trade-off in the allocation of tax revenue between public investment and unemployment subsidy. However, as the allocation ratio of public capital increases, the growth-maximizing tax in the imperfect labor market approaches that of the perfect labor market, and thus reducing the unemployment rate. Finally, we show that the growth-maximizing policy may not be equivalent to the welfare-maximizing policy.

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## 1 Introduction

The purpose of this paper is to examine the effects of policy changes on unemployment and economic growth with public capital in the imperfect labor market. Although many empirical studies of public capital have largely focused on the US economy (see, for example, Aschauer 1989, among others), so far few empirical studies have been attempted on the relationship between public capital and unemployment. Recently, however, a growing literature on countries other than the U.S. has begun to emerge in this field. Using data from selected OECD Economies, Demetriades and Mamuneas (2000) found a negative relationship between public capital as a stock and unemployment in all the selected OECD countries.<sup>1</sup>

It is well known theoretically that public capital enhances economic growth due to an increase in the marginal productivity of private capital presented by Barro (1990). Since Aschauer (1989) showed in their empirical analysis that the stock of public capital is more important than the flow of government spending, the importance of public capital as a stock has been theoretically emphasized by many authors. In endogenous growth models, Futagami et al. (1993), Greiner and Hanusch (1998), and Greiner (1999), among others, have examined the growth and welfare effects of fiscal policies with public capital as a stock variable. However, these studies have typically assumed that the labor market is competitive. To my knowledge, Raurich and Sorolla (2003) provide the only theoretical study investigating the growth effects of public capital as a flow when there is unemployment. They suggested that increasing the amount of public capital may reduce the unemployment rate. Though their analysis amounts to pioneering research in the field of economic growth, it unfortunately relies on a numerical approach. This is partly because they assign an important role to the congestion effect of public capital.<sup>2</sup> The main focus of this paper follows Raurich and

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<sup>1</sup> Demetriades and Mamuneas (2000) also showed that public capital was over-invested in Sweden while it was under-supplied in France and Germany.

<sup>2</sup> So far many authors have extended the Futagami et al. (1993) model in various ways. Glomm and Ravikumar (1994) and Turnovsky (1997) introduced public capital subject

Sorolla (2003) and considers the relationship among public capital, unemployment and growth. One of the features that differentiates our analysis from that of Raurich and Sorolla (2003) is that we follow the conventional public capital model of Futagami et al. (1993). This approach allows us to analytically investigate the balanced growth-maximizing policy and to compare the result in the presence of unemployment with those under the perfect labor market.

It has often been pointed out that high unemployment in most European countries is related to the wage bargaining of trade unions. Since the seminal paper by Daveri and Tabellini (2000),<sup>3</sup> this relationship has been discussed by many authors in an endogenous growth model<sup>4</sup>. However, since their model did not consider both public capital and unemployment simultaneously, it is not applicable to the empirical evidence of Demetriades and Mamuneas (2000). This is the most important point of our study, in which we analyze the effects of fiscal policy on unemployment and economic growth, incorporating the union wage setting into a simple overlapping generations model with the growth engine of public capital. For our purpose, the unemployed are assumed to be supported by unemployment insurance which is financed by both firms and their employees. In addition, we assume here that the government chooses policy variables; the income tax rate and the allocation ratio between public investment and the unemployment subsidy.<sup>5</sup>

The results of this study are as follows. From the optimal rule we derived for the

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to congestion into an endogenous growth model.

<sup>3</sup> Taking account of government spending as divided between the unemployment subsidy and government consumption, Daveri and Tabellini (2000) showed that the rise in unemployment and the reduction in economic growth are caused by an increase in the income tax.

<sup>4</sup> See, for example, Corneo and Marquardt (2000), Bräuninger (2000, 2005), Demmel and Keuschnigg (2000) and Ono (2006a, 2006b), among others.

<sup>5</sup> More recently, OECD (2006) argued how a social transfer system such as unemployment benefits is financed by a part of the income tax and contribution rate in selected OECD countries. In particular, Denmark, Sweden, France and Germany show a similar pattern in that they provide high levels of gross social transfer, though there are considerable differences in the level of the income tax and the contribution rate. While Sweden and Denmark have higher levels of the income tax and the contribution rate, France and Germany collect far less tax and the contribution rate.

maximizing the balanced-growth rate, we show that the firm's contribution to unemployment insurance does not necessarily raise the growth rate. We also show that the relationship between the allocation of tax revenue and the growth rate is an inverted-U shaped. In particular, we demonstrate analytically that the growth-maximizing tax in the imperfect labor market is higher than that of the perfect labor market. However, as the allocation ratio of public capital increases, the growth-maximizing tax in the imperfect labor market approaches that of the perfect labor market, and thus reducing the unemployment rate. Furthermore, we have confirmed that there may exist the welfare-maximizing tax rate that is lower than the growth-maximizing tax rate.

The remainder of this paper is organized as follows. In Section 2 a model is presented, and the steady-state growth path is characterized in Section 3. In Section 4 the growth effects are examined, and in Section 5 the unemployment effects are analyzed. In Section 6 the welfare effects are investigated. Section 7 offers some conclusions.

## 2 Model

We consider an overlapping generations model where individuals live for two periods. Each generation consists of  $N_t$  individuals with  $N_t = (1+n)N_{t-1}$ . Individuals called generation  $t$  are homogenous except for their ages. A representative individual works in the first period, and retires in the second period. The firm produces a single good, which can be consumed or invested.

The preference of any individual in generation  $t$  is given by the utility function  $U_t^i = \ln c_t^i + \beta \ln d_{t+1}^i$  ( $i = w, u$ ), where the superscripts  $w$  and  $u$  denote the employed and unemployed individuals, respectively.<sup>6</sup>  $c_t^i$  and  $d_{t+1}^i$  are the individual's first-period and second-period consumptions, respectively.  $\beta$  is the discount factor

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<sup>6</sup> This setting is similar to that of Corneo and Marquardt (2000) and Josten (2006).

( $0 < \beta < 1$ ).

If the individual is employed ( $i = w$ ), he/she supplies labor to the market and earn wages  $w_t^w$  in the first period. This wage income is allocated to purchasing consumption goods  $c_t^w$ , and savings  $s_t^w$  for future consumption. In the second period, he/she spends all the savings and accrued interest on consumption  $d_{t+1}^w = R_{t+1}s_t^w$ . Thus, the intertemporal budget constraint is:

$$c_t^w + \frac{d_{t+1}^w}{R_{t+1}} = (1 - \tau - \theta_w)w_t^w, \quad (1a)$$

where,  $\tau$  is labor income tax,  $\theta_w$  is the employee's contribution rate to unemployment insurance.

If the individual is unemployed ( $i = u$ ), he/she receives unemployment benefits  $z_t$  in the first period. This benefit is allocated to purchasing consumption goods  $c_t^u$ , and savings  $s_t^u$  for future consumption. In the second period, he/she spends all the savings and accrued interest on consumption  $d_{t+1}^u = R_{t+1}s_t^u$ . Thus, the intertemporal budget constraint is:

$$c_t^u + \frac{d_{t+1}^u}{R_{t+1}} = z_t, \quad (1b)$$

Each household of generation  $t$  maximizes its lifetime utility subject to (1a) (or (1b)). From the optimization conditions, we have the utility-maximizing saving at period  $t$  :

$$s_t^w = \frac{\beta}{1 + \beta} (1 - \tau - \theta_w)w_t, \quad (2a)$$

$$s_t^u = \frac{\beta}{1 + \beta} z_t, \quad (2b)$$

respectively.

Following Barro (1990), we assume that the economy produces a single good according to the following technology:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} g_t^{1-\alpha}, \quad (3)$$

where  $Y_t, K_t$ , and  $L_t$  are the aggregate output, aggregate stocks of private capital, and aggregate labor input in period  $t$ , respectively.  $g_t$  is public services derived from the stock of public capital.  $A$  is the level of technology.

In each period  $t$ , the firm chooses private capital and labor in order to maximize its profit:

$$\pi_t = AK_t^\alpha L_t^{1-\alpha} g_t^{1-\alpha} - R_t K_t - (1+\theta_f)w_t L_t,$$

where  $\theta_f$  is the firm's contribution rate to unemployment insurance. The usual solution to the firm's optimization problem sets factor costs equal to their marginal productivity:

$$R_t = \alpha A g_t^{1-\alpha} \left( \frac{K_t}{L_t} \right)^{\alpha-1}, \quad (4)$$

$$w_t = (1-\alpha)A \frac{1}{1+\theta_f} g_t^{1-\alpha} \left( \frac{K_t}{L_t} \right)^\alpha. \quad (5)$$

The wage level is assumed to be set by the trade union. Denoting the number of those employed by  $L_t$ , since the total population of generation  $t$  is  $N_t$ , the number of those unemployed at  $t$ , is  $N_t - L_t$ . The trade union maximizes the following average income of young generation  $t$ , introduced by Daveri and Tabellini (2000), among others<sup>7</sup>:

$$I_t = (1-\tau-\theta_w)w_t \frac{L_t}{N_t} + z_t \frac{N_t - L_t}{N_t}, \quad (6)$$

subject to (5). The first-order condition is given as:

$$(1-\tau-\theta_w)w_t = \frac{1}{1-\alpha} z_t. \quad (7)$$

Eq (7) means that the union chooses the wage to be a fixed mark-up,  $1/(1-\alpha)$ , on the unemployment benefit.

The government collects a labor income tax and uses the tax revenue for the unemployment subsidy,  $b_t$ , and public capital,  $g_{t+1}$ . Thus the government budget

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<sup>7</sup> See, Ono (2006a) for the interpretation of this specification.

constraint in per worker terms is  $b_t + g_{t+1} = \tau w_t$ . We allow an allocation of tax revenues between  $b_t$  and  $g_{t+1}$  for  $0 < \lambda < 1$ , that is:

$$b_t = (1 - \lambda)\tau w_t, \quad (8)$$

$$g_{t+1} = \lambda\tau w_t. \quad (9)$$

We also assume that the unemployment benefit is composed of unemployment insurance and unemployment subsidies. This system is balanced in each period.

Thus the following equality holds:

$$z_t \frac{N_t - L_t}{N_t} = [(1 - \lambda)\tau + \theta_w + \theta_f] w_t \frac{L_t}{N_t}. \quad (10)$$

From (8) and (10), we can obtain the employment rate:

$$\frac{L_t}{N_t} = \frac{(1 - \alpha)(1 - \tau - \theta_w)}{[(1 - \lambda)\tau + \theta_w + \theta_f] + (1 - \alpha)(1 - \tau - \theta_w)}, \quad (11)$$

Eq (11) is constant and is similar to that of Daveri and Tabellini (2000), among others, while Raurich and Sorolla (2003) derived the employment equation which depends on public capital. From  $N_{t+1} = (1 + n)N_t$  and (11), we can see that  $L_{t+1} = (1 + n)L_t$ .

Finally, the equilibrium condition in the capital market is given as:

$$K_{t+1} = L_t s_t^w + (N_t - L_t) s_t^u. \quad (12)$$

### 3 Equilibrium Growth

We define the balanced-growth rate as  $1 + \gamma = \frac{k_{t+1}}{k_t} = \frac{g_{t+1}}{g_t}$ . Denoting  $k_t$  by the

private capital-labor ratio ( $K_t / L_t$ ), and making use of  $L_{t+1} = (1 + n)L_t$ , (2a), (2b), (5)

and (12), we obtain:

$$(1 + n)k_{t+1} = \frac{\beta}{1 + \beta} (1 - \lambda\tau + \theta_f)(1 - \alpha)A \frac{1}{1 + \theta_f} g_t^{1 - \alpha} k_t^\alpha. \quad (13)$$

Substituting (5) into (9), we obtain:

$$g_{t+1} = \lambda\tau(1-\alpha)A \frac{1}{1+\theta_f} g_t^{1-\alpha} k_t^\alpha. \quad (14)$$

Dividing both sides of (13) by those of (14), we obtain:

$$\left( \frac{k_{t+1}}{g_{t+1}} \right) = \frac{1}{1+n} \frac{\beta}{1+\beta} \frac{1-\lambda\tau+\theta_f}{\lambda\tau}, \quad (15)$$

In equilibrium, the ratio of private capital to public capital used in production in each period is a constant. From (14), and (15), we obtain:

$$1+\gamma = \frac{g_{t+1}}{g_t} = \left( \frac{1}{1+n} \frac{\beta}{1+\beta} \right)^\alpha (1-\alpha)A \frac{(\lambda\tau)^{1-\alpha} (1-\lambda\tau+\theta_f)^\alpha}{1+\theta_f}. \quad (16)$$

There are no transitional dynamics in our model and all variables jump immediately to their steady state values. The equilibrium growth rate depends on the preference parameter,  $\beta$ , the share of physical capital in the production function,  $\alpha$ , and the level of the technology in the production function,  $A$ . It should be noted that a change in the employee's contribution to unemployment insurance is neutral with respect to the economic growth. This result is one of the shared understandings in the literature.<sup>8</sup>

#### 4 Balanced-growth-maximizing fiscal policies

In this section, restricting our concern to the steady-state path, we analyze balanced-growth-maximizing fiscal policies. In particular, we address those fiscal policies by considering the three policy variables, i.e. the share of allocation,  $\lambda$ , the firm's contribution to unemployment insurance,  $\theta_f$ , and the income tax rate,  $\tau$ . From (16), we have the following proposition:

**Proposition 1** When the policy variables, parameterized by  $(\theta_f, \lambda, \tau)$  are satisfied, the following relationship

$$\varphi(\theta_f, \lambda, \tau) \equiv (1-\alpha)(1+\theta_f) - \lambda\tau = 0, \quad (17)$$

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<sup>8</sup> See, for example, Corneo and Marquardt (2000), and Ono (2006a).

the economy maximizes the balanced-growth rate.

**Proof:** See, Appendix 1. □

We now argue the balanced-growth-maximizing policy represented by combinations  $(\tau, \theta_f, \lambda)$ . Consider first the effect of the firm's contribution to unemployment insurance on economic growth. From (17), we obtain the growth-maximizing firm's contribution to unemployment insurance,  $\hat{\theta}_f = \frac{\lambda\tau - (1-\alpha)}{1-\alpha}$ . This result stands in sharp contrast to that of Corneo and Marquardt (2000) who showed that an increase in a firm's contribution to unemployment insurance raises the economic growth. In our model, an increase in  $\hat{\theta}_f$  affects the growth rate in the following two ways. A higher  $\hat{\theta}_f$  leads to more aggregate savings, hence increasing physical capital accumulation. This effect tends to raise the economic growth. On the other hand, there is also a negative effect because an increase in  $\hat{\theta}_f$  reduces investment in public capital which decreases the economic growth.

Under the balanced-growth-maximizing policy, a similar mechanism holds with respect to the allocation ratio,  $\lambda$ . An increase in  $\lambda$  induces the opposite effects on the growth rate. First, a higher  $\lambda$  leads to lower savings, hence reducing physical capital accumulation. This effect tends to decrease economic growth. Second, there is a positive effect because an increase in  $\lambda$  raises investment in public capital, which enhances economic growth. This theoretical result is consistent with recent empirical findings. Many empirical studies have observed a positive growth effect of public investment and other government expenditures (e.g., Devarajan et al., 1996; Kneller et al., 1999; and Shioji, 2001). However, a few studies have found a significantly negative growth effect of public investment in some cases (e.g., Evans and Karras, 1994).

Finally, we turn to analyze the effects of the income tax rate on economic growth. From Proposition 1, we have the following lemma:

**Lemma 1:** If there exists an interior growth-maximizing income tax rate,  $\hat{\tau}$ , it is given as:

$$\hat{\tau} = (1 - \alpha) + \frac{(1 - \alpha)(1 + \theta_f - \lambda)}{\lambda}. \quad (18)$$

It is increasing in  $\theta_f$  and decreasing in  $\lambda$ .

**Proof:** Solving  $d(1 + \gamma)/d\tau = 0$  leads to

$$\hat{\tau} = (1 - \alpha) + \frac{(1 - \alpha)(1 + \theta_f - \lambda)}{\lambda}.$$

Total differentiating  $\lambda$  with respect to  $\tau$  and  $\theta_f$ , respectively:

$$\frac{d\hat{\tau}}{d\theta_f} > 0 \quad \text{and} \quad \frac{d\hat{\tau}}{d\lambda} < 0. \quad \square$$

It should be noted that the growth-maximizing tax in the imperfect labor market is higher than that in the perfect labor market. When  $\lambda = 1$  and  $\theta_f = 0$ , the growth-maximizing tax rate,  $\hat{\tau} = (1 - \alpha)$ , is equal to that derived in Barro (1990) and Futagami et al. (1993), among others. The growth-maximizing tax in the imperfect labor market is expressed by the sum of the following terms. The first term on the RHS of (18) is equal to the elasticity of output with respect to public capital. The second term is the one that generates due to the presence of unemployment. This proposition shows that the presence of unemployment leads to a modification of the fiscal policies when there is full-employment.

The second part of Lemma 1 shows that the share of public capital is crucial for the long-run growth rate of the economy. If the government increases the firm's contribution to unemployment insurance, the growth-maximizing tax in the imperfect labor market can not be closed to that of the perfect labor market. However, if the government increases the allocation ratio of public investment, the growth-maximizing tax in the imperfect labor market approaches that of the perfect labor market.

## 5 The unemployment effects of fiscal policy

In this section, we analyze the effects of fiscal policy on the unemployment rate. From (11), we obtain the unemployment rate:

$$\rho_t = 1 - \frac{L_t}{N_t} = \frac{(1-\lambda)\tau + \theta_w + \theta_f}{[(1-\lambda)\tau + \theta_w + \theta_f] + (1-\alpha)(1-\tau-\theta_w)}. \quad (19)$$

The unemployment rate depends on the tax rate,  $\tau$ , the employee's contribution to unemployment insurance,  $\theta_w$ , the firm's contribution to unemployment insurance,  $\theta_f$ , and the share of physical capital in the production function,  $\alpha$ . The higher the tax rate is, the higher the unemployment rate; the higher the employee's contribution to unemployment insurance is, the higher the unemployment rate; the higher the firm's contribution to unemployment insurance is, the higher the unemployment rate. From (19), we have the following proposition.

**Proposition 2** Shifting public expenditures from the unemployment subsidy to public investment reduces the unemployment rate.

**Proof:** Differentiating (19) with respect to  $\lambda$ , we obtain:

$$\frac{d\rho}{d\lambda} < 0. \quad \square$$

A higher  $\lambda$  leads to a decrease in the wage set by the unions, and hence increases the employment rate (see (11)). This proposition is consistent with empirical studies, such as Pereira and Roca-Sagales (1999) and Demetriades and Mamuneas (2000), who observed the negative relationship between public capital and unemployment.

## 6 The welfare effects of fiscal policy

In this section, we analyze the effects of fiscal policy on the individual's welfare. The steady-state equilibrium level of (average) utility (See Appendix 2 for derivation):

$$W_t = \ln(1 - \tau - \theta_w) + [\beta(\alpha - 1) + \alpha(1 + \beta)] \ln\left(\frac{k}{g}\right) + (1 - \alpha)(1 + \beta) \ln(1 + \gamma)^t. \quad (20)$$

Differentiating (20) with respect to  $\tau$ :

$$\begin{aligned} \frac{dW_t}{d\tau} = & -\frac{1}{1 - \tau - \theta_w} + [(\alpha - 1)\beta + \alpha(1 + \beta)] \left(\frac{k}{g}\right)^{-1} \frac{d\left(\frac{k}{g}\right)}{d\tau} \\ & + t(1 - \alpha)(1 + \beta)(1 + \gamma)^{-1} \frac{d(1 + \gamma)}{d\tau}. \end{aligned} \quad (21)$$

The first term on the RHS of (21) is the effect caused by the increased consumption due to increasing the tax rate. The second term is the effect through the general-equilibrium factor-price changes.<sup>9</sup> We can see that an increase in the tax rate reduces the ratio of private capital to public capital used in production, i.e.,  $d(k/g)/d\tau < 0$ . The third term represents the growth effect, which is given by (18). The welfare effect is expressed by the sum of the above terms. While the first two terms are constant, the last one is non-stationary.

Evaluating (21) at  $\tau = \hat{\tau}$ , we obtain:

$$\left. \frac{dW_t^*}{d\tau} \right|_{\tau=\hat{\tau}} = -\frac{1 + \beta}{1 - \hat{\tau} - \theta_w} - [(\alpha - 1)\beta + \alpha(1 + \beta)] \frac{1 + \theta_f}{\hat{\tau}(1 - \lambda\hat{\tau} + \theta_f)}, \quad (22)$$

From (22), we obtain the following proposition:

**Proposition 3** The share of physical capital in the production function exceeding the discount factor, i.e.,  $\alpha > \beta$ , gives rise to the optimal tax rate,  $\tau^*$ , which is lower than the growth-maximizing tax rate,  $\hat{\tau}$ .

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<sup>9</sup> Introducing this effect, Rangazas (1996) showed that the bequest-constraint affects the growth rate. Yakita (2004) demonstrated that the government's educational subsidies may have a negative effect on economic growth through this effect.

According to de la Croix and Michel (2002, p. 339), the share of physical capital in the production function,  $\alpha$  is set equal to 0.3, and the discount factor,  $\beta$  is equal to  $1/3$ , which is usually calibrated to match the long-run interest rate in models with infinite-lived agents. In this case, the effect through the general-equilibrium factor-price changes term is negative. Thus, the optimal tax rate,  $\tau^*$ , is lower than the growth-maximizing tax rate,  $\hat{\tau}$ . This proposition is similar to that of Futagami et al. (1993) who extended the Barro model by assuming that public capital has a positive effect on aggregate production.

## 7 Conclusion

In this paper, we focus on the fiscal policy implications for unemployment and economic growth in the imperfect labor market. Our purpose is to present a simple growth model with the growth engine of public capital by incorporating the union wage setting. The results of this study are as follows. From the optimal rule we derived for the maximizing the balanced-growth rate, we show that the firm's contribution to unemployment insurance does not necessarily raise the growth rate. We also show that the relationship between the allocation of tax revenue and the growth rate is an inverted-U shaped. In particular, we demonstrate analytically that the growth-maximizing tax in the imperfect labor market is higher than that of the perfect labor market. However, as the allocation ratio of public capital increases, the growth-maximizing tax in the imperfect labor market approaches that of the perfect labor market, and thus reducing the unemployment rate. Furthermore, we have confirmed that the welfare-maximizing tax rate may exist that is lower than the growth-maximizing tax rate.

## Appendix 1: Proof of Proposition 1

Total differentiating (16), we obtain:

$$d(1 + \gamma) = -\frac{\Omega\varphi}{1 + \theta_f} d\theta_f + \Omega\varphi d\lambda + \Omega\varphi d\tau$$

where,

$$\Omega = \left( \frac{1}{1+n} \frac{\beta}{1+\beta} \right)^\alpha (1-\alpha)A \frac{(\lambda\tau)^{1-\alpha} (1-\lambda\tau + \theta_f)^{\alpha-1}}{(1+\theta_f)}$$

$$\varphi(\theta_f, \lambda, \tau) = (1-\alpha)(1+\theta_f) - \lambda\tau$$

respectively. From  $\Omega \neq 0$ , we obtain the relationship,

$$\varphi(\theta_f, \lambda, \tau) \equiv (1-\alpha)(1+\theta_f) - \lambda\tau = 0. \quad \square$$

## Appendix 2: Derivation of (20)

The steady-state equilibrium level of (average) utility is given as:

$$\begin{aligned} W_t &= \frac{1}{N_t} [L_t U_t^w + (N_t - L_t) U_t^u], \\ &= \frac{L_t}{N_t} (\ln c_t^w + \beta \ln d_{t+1}^w) + \frac{N_t - L_t}{N_t} (\ln c_t^u + \beta \ln d_{t+1}^u) \end{aligned} \quad (23)$$

From the optimization conditions, we have:

$$c_t^w = \frac{1}{1+\beta} (1-\tau - \theta_w) w_t \quad (24)$$

$$d_{t+1}^w = \frac{\beta}{1+\beta} R_{t+1} (1-\tau - \theta_w) w_t \quad (25)$$

respectively. Taking  $g_t = g_0(1+\gamma)^t$  into account and from (4), (5), (15), (24) and (25),

the steady-state equilibrium level of the utility of the employed is given as:

$$\ln c_t^w + \beta \ln d_{t+1}^w = \ln \frac{1}{1+\beta} (1-\tau - \theta_w) w_t + \beta \ln \frac{\beta}{1+\beta} R_{t+1} (1-\tau - \theta_w) w_t$$

$$\begin{aligned}
&= (1 + \beta) \ln(1 - \tau - \theta_w) + \beta \ln R_{t+1} + (1 + \beta) \ln w_t + D_1 \\
&= (1 + \beta) \ln(1 - \tau - \theta_w) + \beta \ln \alpha A \frac{1}{1 + \theta_f} g_t^{1-\alpha} \left( \frac{k}{g} \right)^{\alpha-1} \\
&\quad + (1 + \beta) \ln(1 - \alpha) A \frac{1}{1 + \theta_f} g_t^{1-\alpha} \left( \frac{k}{g} \right)^\alpha + D_2 \\
&= (1 + \beta) \ln(1 - \tau - \theta_w) + [\beta(\alpha - 1) + (1 - \alpha)\beta] \ln \left( \frac{k}{g} \right) \\
&\quad + (1 - \alpha)(1 + \beta) \ln g_t + D_3 \\
&= (1 + \beta) \ln(1 - \tau - \theta_w) + [\beta(\alpha - 1) + (1 - \alpha)\beta] \ln \left( \frac{k}{g} \right) \\
&\quad + t(1 - \alpha)(1 + \beta) \ln(1 + \gamma) + D_4. \tag{26}
\end{aligned}$$

where  $D_i$  ( $i = 1, 2, 3, 4$ ) are constant.

Next, from the optimization conditions, we have:

$$c_t^u = \frac{1}{1 + \beta} z_t \tag{27}$$

$$d_{t+1}^u = \frac{\beta}{1 + \beta} R_{t+1} z_t \tag{28}$$

Similarly, taking (7) into account and from (4), (5), (15), (27) and (28), the steady-state equilibrium level of the utility of the unemployed is given as:

$$\begin{aligned}
\ln c_t^u + \beta \ln d_{t+1}^u &= \ln \frac{1}{1 + \beta} z_t + \beta \ln \frac{\beta}{1 + \beta} R_{t+1} z_t \\
&= \ln \frac{1}{1 + \beta} (1 - \alpha)(1 - \tau - \theta_w) w_t + \beta \ln \frac{\beta}{1 + \beta} R_{t+1} (1 - \alpha)(1 - \tau - \theta_w) w_t \\
&= (1 + \beta) \ln(1 - \tau - \theta_w) + \ln R_{t+1} + (1 + \beta) \ln w_t + D_5 \\
&= (1 + \beta) \ln(1 - \tau - \theta_w) + \ln \alpha A \frac{1}{1 + \theta_f} g_t^{1-\alpha} \left( \frac{k}{g} \right)^{\alpha-1} \\
&\quad + (1 + \beta) \ln(1 - \alpha) A \frac{1}{1 + \theta_f} g_t^{1-\alpha} \left( \frac{k}{g} \right)^\alpha + D_6
\end{aligned}$$

$$\begin{aligned}
&= (1 + \beta) \ln(1 - \tau - \theta_w) + [\beta(\alpha - 1) + (1 - \alpha)\beta] \ln\left(\frac{k}{g}\right) \\
&\quad + (1 - \alpha)(1 + \beta) \ln g_t + D_7 \\
&= (1 + \beta) \ln(1 - \tau - \theta_w) + [\beta(\alpha - 1) + (1 - \alpha)\beta] \ln\left(\frac{k}{g}\right) \\
&\quad + t(1 - \alpha)(1 + \beta) \ln(1 + \gamma) + D_8, \tag{29}
\end{aligned}$$

where  $D_i$  ( $i = 5, 6, 7, 8$ ) are constant. Substituting (26) and (29) into (23), we obtain (20). □

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