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Regional Population and Local Public Spending

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Abstract

This paper analyzes the relation between regional population and regional public expenditure by considering the effect of the provision of public services. In the analysis, we consider the possibility of perfect agglomeration: that only one region exists and that other regions have disappeared. Moreover, the paper analyzes the other case that each region always exists because of fixed regional boundaries.

In efficient allocation, if the cost per capita of a local public good declines with population size, public expenditure per capita is higher in the more populated region even though the amount of public goods is smaller. In the larger region, because of the cost effect, public expenditure is larger. Conversely, if the cost per capita increases with the population, the amount of local public goods increases with the population size. However, in this case, the full agglomeration is efficient and only one region exists.

When regional boundaries are fixed, results change only when the cost per capita function does not drastically increase. In other words, because of the demand effect, public expenditure per capita might be higher in the region with the larger population even though this case does not arise in efficient allocation: this case is realized in a fixed territory though.

JEL classification: R51, H72, R23, H73

Keywords: local public expenditure; regional population; local public goods

1 Introduction

This paper analyzes the relation between local public spending and regional population. For example, municipal annexation is regarding how municipal governments add to the population. (For example, Dur and Staal (2008), Edwards (2011), and Durst (2014) analyzes the annexation.) Consider that the object of annexation is fiscal benefit, such as a greater tax base. If public spending increases with the population, this effect conflicts with fiscal benefit. Therefore, the relation between local public spending and regional population affects annexation activity.

Several studies note that the regional population affects the cost of public services. Buettner, Schwager and Stegarescu (2004) show the positive association between cost and population. However, these studies do not consider the association between the provision of public services and the population. If the provision of public services changes with the population, it affects local public spending. For example, Bates and Santerre (2013) analyzes the local public health services. By considering the cost and provision of the local public good, this paper analyzes the relation between local public spending and regional population.

Buettner and Holm-Hadulla (2013) examine the relation between local public spending and regional population in an efficient allocation. In this analysis, the case that local public expenditure per capita is higher in more populated regions exists because the demand for local public goods increases with regional population. However, in analyzing efficient allocation, they do not consider the case that all population agglomerates in one region. In a spatial economy, private industries cause various externalities that induce agglomeration. For example, Behrens and Murata (2007, 2009) investigate optimal resource allocation under monopolistic competition. It is possible that all population agglomerates in one region as the optimal allocation. Assuming that agglomeration might arise, Lee and Choe (2012) evaluate local government behavior.

Buettner and Holm-Hadulla (2013) do not consider the case of a fixed territory. In efficient allocation, land is freely distributed across regions, which is possible when regional boundaries can be freely changed. However, in reality this is very difficult because the distribution of land is fixed. This paper analyzes this case.

Specifically, this paper looks at the relation between regional population and regional public expenditure. In the analysis, we consider the possibility of perfect agglomeration, where all population agglomerates in one region. Moreover, the paper analyzes the other case: when the allocation of land is fixed.

This paper is organized as follows. Section 2 introduces the model and analyzes a case of efficient allocation. Section 3 considers the case of fixed regional boundaries. Section 4 concludes this study.

2 Efficient Allocation in a Regional Economy

This paper's model follows Buettner and Holm-Hadulla (2013), who consider an economy with two regions. The population in region i $(i = 1, 2)$ is n_i and the total population is $\bar{N} = n_1 + n_2$. Individuals can migrate across regions without cost. Each individual supplies one unit of labor.

Each region's land area is H_i and the total land area $H = H_1 + H_2$. These land areas are freely distributed across regions, which alters regional boundaries. The land is used for housing and land, which in each region is distributed equally among individuals. The amount of land consumed for housing per capita is H_i/n_i.

The private good is produced with the labor as the input. It is the numeraire good. In region i, one worker can produce β_i units of a private good. It is assumed that $\beta_1 > \beta_2$. That is, region 1 is more productive than other regions. The private good is utilized for consumption and is the production factor for the local public good.

The local public good is supplied in each region. The amount of private good needed to produce g_i units is ${g_i}^{\gamma}$ where γ is the degree of scale economy. The production of public good is increasing returns to scale when $\gamma < 1$ and decreasing returns to scale when $\gamma > 1$. The local public good in region i can be consumed by only region i's residents. Consumption of the local public good z_i is determined by the amount of the local public good g_i and the population n_i , formally $z_i = n_i^{-\delta} g_i$ where δ is the congestion elasticity. In the case of a pure local public good, $\delta = 0$. The cost function of a local public good $C(z_i, n_i)$ is derived as ${z_i}^{\gamma}{n_i}^{\gamma\delta}$.

The central planner aims to maximize a representative individual's utility. Individuals in region i obtain utility from the consumption of the private good x_i , the local public good z_i and the land h_i. The utility function is

$$U_i = \log x_i + \log z_i + \log h_i$$

The resource constraint in the economy is as follows:

$$\beta_1 n_1 + \beta_2 n_2 = n_1 x_1 + n_2 x_2 + C(g_1, n_1) + C(g_2, n_2)$$

where x_i is the consumption of the private good. When the utility u and the local public good z_i are given, $x_i = e^u \frac{n_i}{z_i H_i}$. From this fact and the cost function of the local public good, the resource constraint is :

$$\beta_1 n_1 + \beta_2 n_2 = {n_1}^2 \frac{e^u}{z_1 H_1} + {n_2}^2 \frac{e^u}{z_2 H_2} + {z_1}^{\gamma}{n_1}^{\gamma\delta} + {z_2}^{\gamma}{n_2}^{\gamma\delta}$$

The resulting Lagrangean is as follows:

$$L \equiv u + \lambda \left[\beta_1 n_1 + \beta_2 (N - n_1) - {n_1}^2 \frac{e^u}{z_1 H_1} - (N - n_1)^2 \frac{e^u}{z_2 (H - H_1)} - {z_1}^{\gamma}{n_1}^{\gamma\delta} - {z_2}^{\gamma}(N - n_1)^{\gamma\delta} \right]$$

First-order conditions for n_1, z_1, z_2 and H_1 are as follows:

$$\beta_1 - \beta_2 - \frac{2 n_1 e^u}{z_1 H_1} + \frac{2(N - n_1)e^u}{z_2 (H - H_1)} - \gamma\delta {z_1}^{\gamma}{n_1}^{\gamma\delta - 1} + \gamma\delta {z_2}^{\gamma}(N - n_1)^{\gamma\delta - 1} = 0 \quad (1)$$

$$\frac{{n_1}^2 e^u}{{z_1}^2 H_1} - \gamma {z_1}^{\gamma - 1}{n_1}^{\gamma\delta} = 0 \quad (2)$$

$$\frac{(N-n_1)^2 e^u}{{z_2}^2(H-H_1)} - \gamma {z_2}^{\gamma-1}(N-n_1)^{\gamma\delta} = 0 \tag{3}$$

$$\frac{e^u {n_1}^2}{z_1 {H_1}^2} - \frac{e^u (N-n_1)^2}{z_2(H-H_1)^2} = 0 \tag{4}$$

Following Buettner and Holm-Hadulla (2013), equation (1) is the locational efficiency condition. Equations (2) and (3) are about efficient provision of the local public good, which are Samuelson conditions. Equation (4) states the optimal allocation of land among two regions.

From these equations, the efficient allocation of land is indicated, giving the following equation:

$$\frac{H_1}{H-H_1} > \frac{n_1}{N-n_1}$$

Interpretation of the equation says that region 1's share of land is larger than its share of population in an efficient allocation. For the population, the following condition is derived:

$$\left(\frac{n_1}{N-n_1}\right)^{\frac{\gamma\delta-1}{2\gamma+1}} > 1$$

When $\gamma\delta < 1$, the population of region 1 is smaller than that of region 2. That is, when the cost of local public good per capita $\frac{C(z_i,n_i)}{n_i} = z_i^{\gamma} n_i^{\gamma\delta-1}$ declines with population size, region 1's population is smaller. The interpretation of this result is as follows. If the population of region 1 is larger, the cost of the local public good is smaller. Then, because the amount of the local public good is larger, all residents want to migrate to region 1. To accomplish the equilibrium allocation, it is necessary that the population size of region 2 be larger. In this allocation, a larger amount of the local public good can be provided in region 2 and the level of utility of all residents is equalized. Conversely, when $\gamma\delta > 1$ region 1's population size is larger. In this case, the amount of the local public good in region 2 is greater and efficient allocation is achieved.

From first-order conditions, the utility in that allocation is as follows:

$$u_D = \frac{\gamma+1}{\gamma} \log \frac{\{\beta_1 n_1^* + \beta_2(N-n_1^*)\} H^{\frac{\gamma}{\gamma+1}}}{\frac{\gamma+1}{\gamma}\gamma^{\frac{1}{\gamma+1}} \left\{ n_1^{*\,\frac{2\gamma+\gamma\delta}{2\gamma+1}} + (N-n_1^*)^{\frac{2\gamma+\gamma\delta}{2\gamma+1}} \right\}^{\frac{2\gamma+1}{\gamma+1}}} \tag{5}$$

In this economy, it is possible that all land area is distributed in one region. At the point of efficient allocation, this corner solution is efficient if the utility is larger than in the case of an inner solution. When region 1 receives all land areas, all population agglomerates in region 1. In this full agglomeration, the utility is as follows:

$$u_A = \frac{\gamma+1}{\gamma} \log \frac{\beta_1 N H^{\frac{\gamma}{\gamma+1}}}{\frac{\gamma+1}{\gamma}\gamma^{\frac{1}{\gamma+1}} \left\{ N^{\frac{2\gamma+\gamma\delta}{2\gamma+1}} \right\}^{\frac{2\gamma+1}{\gamma+1}}} \tag{6}$$

Comparing u_D and u_A , if

$$\frac{\beta_1 n_1^* + \beta_2(N-n_1^*)}{\beta_1 N} > \left[\frac{n_1^{*\,\frac{2\gamma+\gamma\delta}{2\gamma+1}} + (N-n_1^*)^{\frac{2\gamma+\gamma\delta}{2\gamma+1}}}{N^{\frac{2\gamma+\gamma\delta}{2\gamma+1}}} \right]^{\frac{2\gamma+1}{\gamma+1}}$$

the inner solution that each region receives the land is optimal. This is possible only if $\gamma\delta > 1$, that is, it is efficient that each region exists. Conversely, if $\gamma\delta < 1$, $u_D < u_A$, and the full agglomeration is efficient. In the following, we analyze efficient allocation of the local public good in the case that each region exists. That is, $\gamma\delta > 1$ is satisfied. From the first-order condition, the following equation is derived.

$$\frac{z_1}{z_2} = \left[\frac{H - H_1}{H_1}\frac{n_1}{N - n_1}\right]^2 \tag{7}$$

Because the region 1's share of land is larger than its share of the population, $z_1/z_2 < 1$. Moreover, when $\gamma\delta > 1$, region 1's population is greater. That is, a larger amount of the local public good is provided in the less populated region. The cost of the local public good per capita is $C(z_i, n_i)/n_i = z_i^{\gamma} n_i^{\gamma\delta - 1}$. From the first-order condition, the following condition holds.

$$\frac{C(z_1, n_1)/n_1}{C(z_2, n_2)/n_2} = \left(\frac{n_1}{n_2}\right)^{\frac{\gamma\delta - 1}{2\gamma + 1}} \tag{8}$$

Because $\gamma\delta > 1$ and $n_1 > n_2$, this equation is larger than 1. That is, the public expenditure per capita is larger in region 1. Therefore, the following proposition is derived.

Proposition 1 If $\gamma\delta < 1$, the full agglomeration that only one region exists is efficient. Only if $\gamma\delta > 1$, is it possible that the inner allocation is efficient.

In the inner allocation, the higher productive region has a larger population and a smaller amount of the public good. Moreover, the level of public expenditure per capita is higher in the larger region.

When the inner allocation is efficient, the region with larger population has a smaller local public good even though the public expenditure per capita is higher. Because the congestion effect is larger in the production of the local public good, and the scale economy is smaller, the public sector cost is higher in the region even though the amount of the local public good is smaller. This means that demand for the local public good does not increase public expenditure. Public expenditure per capita is higher in the region with a larger population because of higher cost, not because of public demand.

When $\gamma\delta < 1$, it is not efficient for each region to exist. If this inner equilibrium is realized, the population of the less productive region is larger than the other. Then, in that region the amount of the local public good is larger, even though the public expenditure per capita is lower. This means that the demand for the local public good does not increase the public expenditure per capita even though that demand increases with population.

3 Efficiency in the Case of a Fixed Territory

The previous section shows that the public expenditure per capita is higher in the more populated region because of the cost effect. Moreover, the demand for local public goods does not increase with population size in an efficient allocation. In that analysis, land can be efficiently allocated across regions. However, in reality, regional boundary changes are difficult, and each region has a fixed territory. This section analyzes the case when the land size of each region is fixed. It is assumed that each region has the same amount of land area, that is, $H_1 = H_2 = H/2$.

Following the previous section, the central planner's Lagrangean is as follows:

$$L' \equiv u + \lambda \left[\beta_1 n_1 + \beta_2(N - n_1) - {n_1}^2 \frac{2e^u}{z_1 H} - (N - n_1)^2 \frac{2e^u}{z_2 H} - {z_1}^\gamma {n_1}^{\gamma\delta} - {z_2}^\gamma (N - n_1)^{\gamma\delta}\right]$$

First-order conditions for n_1, z_1, z_2 are as follows:

$$\beta_1 - \beta_2 - \frac{4n_1 e^u}{z_1 H} + \frac{4(N - n_1)e^u}{z_2 H} - \gamma\delta {z_1}^\gamma {n_1}^{\gamma\delta - 1} + \gamma\delta {z_2}^\gamma (N - n_1)^{\gamma\delta - 1} = 0 \quad (9)$$

$$\frac{2{n_1}^2 e^u}{{z_1}^2 H} - \gamma {z_1}^{\gamma - 1} {n_1}^{\gamma\delta} = 0 \quad (10)$$

$$\frac{2(N - n_1)^2 e^u}{{z_2}^2 H} - \gamma {z_2}^{\gamma - 1} (N - n_1)^{\gamma\delta} = 0 \quad (11)$$

Equation (9) is the locational efficiency condition. Equations (10) and (11) are about the efficient provision of the local public good.

Regarding the efficient allocation of the population, the following condition is derived:

$$\beta_1 - \beta_2 - (2 + \delta)\gamma^{\frac{1}{\gamma+1}} (e^u)^{\frac{\gamma}{\gamma+1}} \left(\frac{H}{2}\right)^{\frac{-\gamma}{\gamma+1}} \left[{n_1}^{\frac{\gamma\delta+\gamma-1}{\gamma+1}} - (N - n_1)^{\frac{\gamma\delta+\gamma-1}{\gamma+1}}\right] = 0 \quad (12)$$

Because $\beta_1 > \beta_2$, when $\gamma\delta + \gamma - 1 > (<) 0$, $n_1 > (<) n_2 = N - n_1$. $\gamma\delta + \gamma$ is the degree of the homogeneous function for the cost of the local public good. From $C(z, n) = z^\gamma n^{\gamma\delta}$, if $\gamma + \gamma\delta = 1$, $C(tz, tn) = tC(z, n)$ and the cost for the local public good is a homogeneous function of degree 1. Therefore, when the degree of this homogeneous function is larger than 1, the population in region 1 is larger than region 2. Because public sector costs increase quickly with population, it is not efficient that the less productive region has the higher population. Conversely, when the degree of that homogeneous function is smaller than 1, the population in region 1 is smaller. In this case, the low-productive region can sustain a larger population in efficient allocation.

To show the efficient allocation of the local public good, let us consider the first-order condition for the local public good. Then, the following equation is derived:

$$\frac{z_1}{z_2} = \left(\frac{n_1}{N - n_1}\right)^{\frac{2-\gamma\delta}{\gamma+1}} \tag{13}$$

If $\gamma\delta > 2$, $\gamma\delta + \gamma - 1 > 0$, then, from the condition for the efficient allocation of the population, $n_1 > n_2$ and $z_1 < z_2$. Conversely, if $\gamma\delta < 2$, the amount of the local public good is larger in the region with the larger population. Moreover, for the expenditure per capita of the local public good, the following condition holds true:

$$\frac{C(z_1, n_1)/n_1}{C(z_2, n_2)/n_2} = \left(\frac{n_1}{n_2}\right)^{\frac{\gamma\delta+\gamma-1}{\gamma+1}} \tag{14}$$

From the perspective of the efficient allocation of the population, the expenditure per capita is larger in region 1. To sum up, the following proposition is derived.

Proposition 2 Assume that land is equally distributed across regions. If $\gamma\delta > 2$, the more productive region has a larger population and level of public expenditure per capita even though the amount of the local public good is smaller. Conversely, if $\gamma\delta < 2$, in the region with a larger population the amount of the local public good is larger even though the more productive region has a larger level of public expenditure per capita.

$\gamma\delta > 2$ means that the cost per capita function increases and that the curve is U-shaped with population size. For example, the human services, such as health care and education, have that cost function. Then, in the higher productive region, population and expenditure per capita is larger even though the amount of the local public good is smaller. This result occurs because the cost effect is larger in the production of the local public good. In the smaller region, it is efficient that the amount of the local public good is larger.

Conversely, if the cost per capita function does not drastically increase ($\gamma\delta < 2$), the amount of the local public good is bigger in the region with the larger population. On the other hand, whether or not a productive region has larger population size, the expenditure per capita is larger in that region. As a result, it is possible that the public expenditure per capita increases with population size because of the demand effect. That is, when the degree of the homogeneous cost function is larger than 1 ($\gamma\delta + \gamma - 1 > 0$), the more populated region increases its demand for local public goods, and that demand increases the public expenditure per capita. In efficient allocation, this case does not arise, because demand does not increase with population.

4 Conclusion

This paper examines the relationship between regional population and regional public expenditure by considering the effect of the provision of local public goods. Buettner and Holm-Hadulla (2013) show that the efficient level of public expenditure is higher in the larger region because of the demand effect. However, they do not consider the case of a fixed territory. Moreover, they do not consider the possibility of full agglomeration in efficient allocation. When production asymmetry exists, it may be efficient for all factors of the product to agglomerate in one region. In the analysis of that relation, this paper considers these effects.

If the cost per capita of the local public good declines with the population, the public expenditure per capita is higher in the more populated region even though the amount of the public good is smaller. In the larger region, because of the cost effect, the public expenditure is greater. Conversely, if the cost per capita increases with population, the amount of the local public good increases with population as well. However, in this case, full agglomeration is efficient and only one region exists.

When the region exists because of fixed allocation of land, the result only changes when the cost per capita function does not drastically increase. In other words, because of the demand effect, public expenditure per capita may be higher in the region with the larger population. Buettner and Holm-Hadulla (2013) show a case demonstrating this result. However, this paper's analysis shows that this case is realized in a fixed territory even though this case does not arise in efficient allocation.

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