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Abstract

In this study, we considered the role of intercity dependence in the era of depopulation, where congestion matters little for some regions. Using a simple two-city migration model with consumer optimization, we identified three factors that support intercity dependence: complementarity between services in different cities, preference of consumers for consumption when travelling, and the quality of intercity transportation. If these three factors adequately support intercity dependence, we found that two cities can permanently coexist in equilibrium even if there is a significant population loss at the aggregate level.

JEL Classification Codes: O18, P25, R12

Keywords: Multiple cities; optimal city size; interdependence; transportation; population decline

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1 Introduction

Congestion has long been a key issue for researchers and policy makers concerned with urbanization. Previous analyses of the formation of systems of cities were generally based on the negative external effects of congestion on the performance of an urban area, such as its productivity. For instance, seminal studies based on the Henry George Theorem, a well-known result in urban economics, showed that the optimal city population size is achieved when: "the agglomerative economies of scale that are responsible for the city's formation are balanced by diseconomies of scale" (Arnott 1998) typically caused by congestion. According to the interaction between these opposite forces, studies such as that by Kanemoto (1980) investigated the composition of cities in a region (e.g., the optimal number of cities) and its welfare implications. However, the speed of population growth has been slowing recently in some developed economies due to aging. Thus, depopulation is becoming an increasingly prominent issue for these aging economies,[1] where congestion will be naturally resolved as a serious urban problem.

How does the composition of cities change when congestion matters little? Does it inevitably converge to a single city environment or is it possible for multiple cities to survive perpetually? In this study, we address these questions by focusing on intercity dependence as an essential determinant of the composition of cities in a congestion-free depopulated environment.

We develop a simple two-city model of migration with consumer optimization by considering the following three features. To incorporate intercity dependence into the model, we assume the following. (a) People consume differentiated services provided in both cities, where residents in one city consume their own city's service but also those of the other city by using intercity transportation. We assume an iceberg cost when people travel to consume the other city's service. (b) Each city's attractiveness (or its service quality) is determined by its population size according to two standard forces comprising the agglomeration economies and diseconomies. (c) Finally, to consider a sufficiently depopulated region, we assume that the regional population size has been sufficiently depopulated such that the negative effect of congestion is sufficiently small. Thus, in this region, the city residents always benefit from an increase in the size of their city's population. Even in this simple setting, we show that the optimal/equilibrium population size of a city is not as obvious as it seems.

Our main finding is that the relationship between a city's population size and its residents' welfare (measured by indirect utility) can be an inverted U-shape in our model when the services provided by two cities are relatively complementary and intercity transportation also exists. Therefore, even in a sufficiently depopulated region where the agglomerative economies of scale always dominate diseconomies of scale (by congestion), an optimal city population size can exist if the interdependence between cities is deep in the sense defined above. This implies that the complementary interdependence between cities might be another factor that affects the optimal city population size, in addition to the well-established factor comprising the balance between scale economies and diseconomies.

Under free migration, we also show that two cities can coexist permanently in our model where scale diseconomies do not explicitly operate due to the interdependence between cities. In addition to the complementarity between cities, we identify two more factors that support interdependence: a preference for consumption by travelling and the quality of intercity transportation. In particular, the preference for travelling consumption is always conducive to the equilibrium with multiple cities, but the effect of the quality of transportation is ambiguous. The quality of intercity transportation can allow more substitutable cities to persist in equilibrium but the same does not apply to relatively complementary cities. Better transportation might

[1] In particular, some metropolitan areas have been significantly depopulated within a relatively short period of time (Pallagst 2009). The infrastructure and administrative systems in these shrinking cities were originally designed to support a huge population and to mitigate congestion, so the reforms needed to retain (and enhance) the quality of life for the remaining residents in shrinking cities are an important new issue.

not always support the existence of multiple cities.

It should also be noted that our results do not depend essentially on the size of the regional population (which is assumed to be given in the analysis). Thus, in our model, depopulation is a problem because the negative effects of congestion disappear when the total population of the region is excessively small. However, the speed of depopulation or the absolute level of the total population is not as critical regardless of whether multiple cities can permanently coexist or not.

Our basic research question and modeling strategy originate from the seminal contributions to the golden rule for local public finances defined by the Henry George Theorem.[2] Our approach differs from those employed in previous studies in main ways. First, we only assume agglomerative economies of scale to consider the depopulation situation. Second, we introduce and explore the role of interdependence between cities and how the quality of intercity transportation contributes to interdependence. Third, to facilitate these two extensions, we consider a pure consumption economy in the main analysis, although we present a general equilibrium version with both consumption and production in the Appendix. According to these three considerations, we provide a novel approach to urban economics.

We model interdependence by applying the method proposed by Murata (2002) for modeling rural-urban interdependence to our environment containing two basically symmetric cities. By using a constant elasticity of substitution function, the substitutability/complementarity between two places captures the degree of interdependence between the cities. This method allows us to characterize a new role for *intercity* dependence in the optimal and equilibrium city population size under the existence of free migration. We extend the method given by Murata (2002) by introducing the iceberg travelling cost, thereby allowing us to analyze the quality of transportation between interdependent areas (i.e., cities in our model).

2 Model of Differentiated Cities with Intercity Transportation

Consider a region with a population N. The regions contains two cities, A and B. Each city contains residents L_i with $i = A, B$, who can freely migrate between the cities but they cannot fly out of the region. We note that $N = L_A + L_B$. Then, we suppose that each city is differentiated in the sense that it has unique cultural and/or industrial characteristics. People have separate preferences for these cities.

2.1 Preferences

We consider that people consume city services, which can potentially include commercial goods/services and public/private attraction points (e.g., museums, gardens, or nature outdoors). The unit price for services in city i, p_i, is paid in terms of a numeraire good.

A distinct feature of the model is the lack of a production factor. Thus, consider a so-called pure consumption economy, where the initial endowment of the numeraire good for a resident in city i, m_i, is exogenously given. We label m_i as an income. Our aim when abstracting the production factors is to highlight the role of intercity dependence in city population sizes in the case where a region is sufficiently depopulated. This is because the simple pure consumption setting presented in the following is sufficient for illustrating the results. However, we also consider an extended model with production and general equilibrium. In particular, we show that our results are not essentially changed by these extensions (see the Appendix for details).

[2] See Flatters et al., (1974), Henderson (1977), Stiglitz (1977), Arnott and Stiglitz (1979), Kanemoto (1980), and Schweizer (1983). In addition, see Behrens, Kanemoto, and Murata (2015) for a more recent contribution.

To model interdependence between cities, we employ the method proposed by Murata (2002) for modeling rural-urban interdependence in the present study of intercity dependence. We assume that city services are differentiated between the cities. To describe the nature of interdependence, as suggested by Murata (2002), we assume a standard constant elasticity of substitution utility function. In particular, residents in city A are endowed with the following utility function:

$$U_A = \left[(\alpha_A c_A)^{\frac{\varepsilon-1}{\varepsilon}} + \lambda(\alpha_B \tilde{c}_{AB})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where they consume c_A units of their own city's services and $\tilde{c}_{AB}$ units of city B's services. We use a_i to denote the attractiveness of city i (or its services provided) and $\lambda > 0$ is a preference parameter for "consumption with travelling." In a hypothetical situation where $\lambda = 0$, the utility function becomes $U_A = \alpha_A c_A$, which is essentially the same as the case where intercity transportation is not available.

2.2 Intercity Transportation

To define the role of intercity transportation in our model, we first note that residents in city A are under the following budget constraint:

$$m_A = p_A c_A + p_B c_{AB}, \tag{2}$$

where c_{AB} denotes the amount that a resident in city A purchases in city B. A key assumption in the present study is that the purchase c_{AB} and consumption $\tilde{c}_{AB}$ generally differ because agents experience disutility from traveling due to their time used for moving. We assume that:

$$\tilde{c}_{AB} = (1 - \tau_{AB})\, c_{AB}, \tag{3}$$

where $0 < \tau_{AB} < 1$ is a parameter that captures the extent to which agents in city A experience disutility from moving to city B. The purchase c_B is depreciated more strongly with a higher τ_{AB}, and the travelling costs are very high when $\tau_{AB} \to 1$. We define $q_{AB} \equiv 1 - \tau_{AB} \in (0, 1)$, which can be labeled somewhat imprecisely as the quality of intercity transportation for residents in city A. This labeling shows that high quality transportation facilitates travelling and mitigates the disutility by reducing the travelling/waiting time and/or enhancing the comfort during a trip. Thus, the quality of intercity transportation is higher for residents in city A when q_{AB} is greater.

A notable feature of our model is the asymmetric quality of transportation in different directions, which captures an important aspect of intercity transportation. In reality, the demand for the use of intercity transportation varies in terms of the time and direction, even within a single day. Thus, a schedule for trains and buses is often optimized to meet the peak demand *only in a single direction* due to the asymmetric demands between directions. Thus, the quality of intercity transportation from city A to B and that from city B to A may be different in some cases. We can also observe this type of asymmetry between directions in traffic signals. In our model, different values for q_{AB} and q_{BA} describe the asymmetry of the transportation quality between different directions.

2.3 Optimization and Indirect Utility Functions

The maximization problem can be written as $\max_{\{c_A, \tilde{c}_B, c_B\}} U_A$ subject to (2) and (3). Given that consumers are price takers, we solve the problem to obtain the demand functions as:

$$c_A = \frac{(p_A/\alpha_A)^{1-\varepsilon}}{(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon (p_B/(q_{AB}\alpha_B))^{1-\varepsilon}} \frac{m_A}{p_A} \text{ and } c_{AB} = \frac{\lambda^\varepsilon (p_B/(q_{AB}\alpha_B))^{1-\varepsilon}}{(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon (p_B/(q_{AB}\alpha_B))^{1-\varepsilon}} \frac{m_A}{p_B}. \tag{4}$$

The indirect utility function is then derived as:

$$U_A^I = \frac{m_A}{\left[(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon \left(p_B/(q_{AB}\alpha_B)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}. \tag{5}$$

Due to symmetry, we can also obtain:

$$c_B = \frac{(p_B/\alpha_B)^{1-\varepsilon}}{\lambda^\varepsilon \left(p_A/(q_{BA}\alpha_A)\right)^{1-\varepsilon} + (p_B/\alpha_B)^{1-\varepsilon}} \frac{m_B}{p_B} \text{ and } c_{BA} = \frac{\lambda^\varepsilon \left(p_A/(q_{BA}\alpha_A)\right)^{1-\varepsilon}}{\lambda^\varepsilon \left(p_A/(q_{BA}\alpha_A)\right)^{1-\varepsilon} + (p_B/\alpha_B)^{1-\varepsilon}} \frac{m_B}{p_A}, \tag{6}$$

$$U_B^I = \frac{m_B}{\left[\lambda^\varepsilon \left(p_A/(q_{BA}\alpha_A)\right)^{1-\varepsilon} + (p_B/\alpha_B)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}. \tag{7}$$

In the next section, we use these expressions to characterize the optimal and equilibrium city size in the depopulated region.

3 Interdependence and Optimal City Population Size in a Depopulated Region

Next, we characterize the optimal city size in a sufficiently depopulated region, where we focus on two external effects of population size as a determinant of city attractiveness, α_i, comprising a positive externality of agglomeration and a negative externality of congestion. Due to these two opposite externalities, according to the standard approach employed in previous studies, the productivity in city i (taking a form of city attractiveness α_i in our model) is an inverted U-shaped function of the city population size, L_i. Denote $\alpha_i \equiv \alpha(L_i)$ with $\alpha' > 0$ for $L_i < l_0$ and $\alpha' \leq 0$ for $L_i \geq l_0$, where l_0 is the cutoff level under (above) which the positive (negative) effect of population size dominates, and this is point at which the agglomerative economies of population size (scale) are balanced by diseconomies due to congestion.

Let us impose the following assumption for a depopulated region.

Assumption 1 (Depopulated Region) *The regional population N has become sufficiently depopulated such that $N < l_0$ holds. Thus, the negative effect of congestion does not matter and the positive agglomeration externality always dominates on the feasible domain.*

Under Assumption 1, the attractiveness of city i, $\alpha(L_i)$, always increases with its population L_i; thus, $\alpha(L_i)' > 0$ holds for the feasible domain of L_i, i.e., $[0, N]$ (note that $N < l_0$). In this setting, the indirect utility function for residents in city i may simply be a monotonically increasing function of the population of city i because the negative effect does not operate explicitly. By contrary, we demonstrate that the indirect utility function can even be an inverted U-shaped function in our setting because the deeper interdependence between the two cities is supported by the existence of intercity transportation. In the following, for simplicity, we consider a linear externality function, $\alpha_i = L_i$.

Proposition 1 *Under Assumption 1, the city attractiveness α_i increases monotonically with the population size L_i. However, the indirect utility function, U_i^I, is an inverted U-shaped function of the city size, L_i, when the interdependence between cities is sufficiently connected such that the elasticity ε of substitution is smaller than 2. The optimal level of the city size is given as:*

$$L_i^* \equiv \frac{N}{1 + \lambda^{\frac{\varepsilon}{2-\varepsilon}} \left(q_{ij}p_i/p_j\right)^{\frac{\varepsilon-1}{2-\varepsilon}}}, \tag{8}$$

in which $i \in \{A, B\}$, $j \in \{A, B\}$, $i \neq j$.

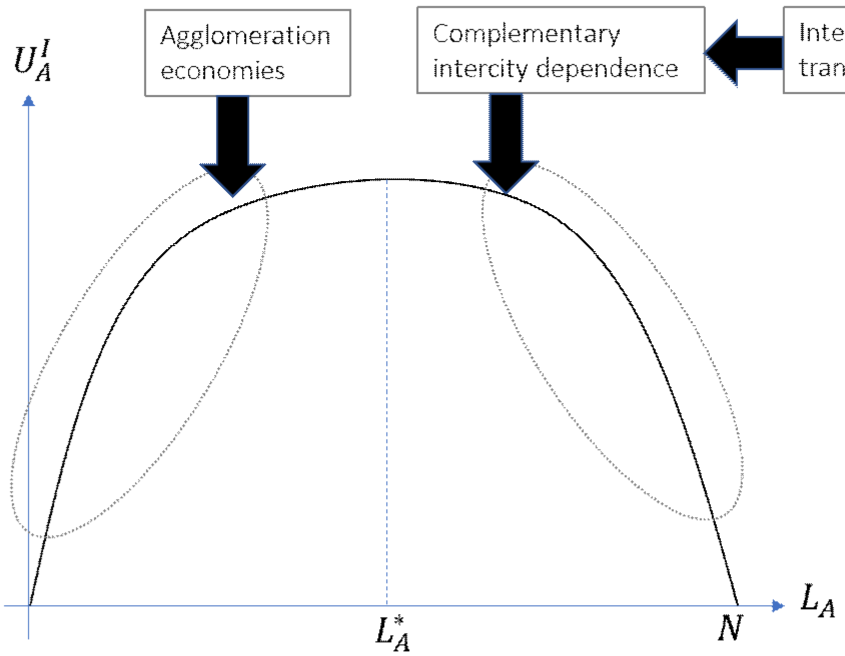

Figure 1: City size and indirect utility for $\varepsilon < 2$

Proof. By differentiating (5) with respect to L_A, we obtain

$$\frac{dU_A^I}{dL_A} = \frac{L_A^{\varepsilon-2} - \lambda^\varepsilon (q_{AB}p_A/p_B)^{\varepsilon-1}(N-L_A)^{\varepsilon-2}}{(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon (p_B/(q_{AB}\alpha_B))^{1-\varepsilon}} (p_A)^{1-\varepsilon} m_A, \tag{9}$$

where we have used $\alpha_i = L_i$ and $L_A + L_B = N$. By (9), $dU_A^I/dL_A > 0$ if and only if

$$\left(\frac{L_A}{N-L_A}\right)^{\varepsilon-2} > \lambda^\varepsilon (q_{AB}p_A/p_B)^{\varepsilon-1}. \tag{10}$$

When $\varepsilon < 2$, the left-hand side of (10) is decreasing in L_A, from $+\infty$ for $L_A \to 0$ to 0 for $L_A \to N$. Thus, $dU_A^I/dL_A > (<)0$ holds if $L_A < L_A^*$, where L_A^* is defined in (8). Due to symmetry, the proof can be completed easily. ■

Under Assumption 1, the entire region is so small that the negative congestion effect does not appear to operate explicitly. If there is no intercity transportation, the consumers in one city cannot consume anything in the other city. Thus, the utility function would become $U_i = \alpha_i c_i$, and the indirect utility function would increase monotonically in population L_i due to the positive externality effect dominating all over the feasible domain, $L_i \in [0, N]$.

Somewhat counterintuitively, Proposition 1 proves that even in this environment, the indirect utility function can have a negatively sloped part under the existence of intercity transportation. Intercity transportation supports the interdependence between cities by enabling intercity consumption. If the two cities are sufficiently complementary, the welfare (measured by indirect utility) is a single-peaked function of the population, thereby implying the existence of an optimal city population size.

Why does an optimal population size exist even if the region is free from congestion effects? The answer is quite simple and it is the main focus of our study. Without any loss of generality, we focus on city A. First, the attractiveness α_A increases with the population size L_A under Assumption 1, which creates a positive direct effect of L_A on U_A^I. Second, an increase in L_A denotes a decrease in the population size of city B, $L_B = N - L_A$, which makes city B less attractive (i.e., a decrease in α_B). Thus, this creates an indirect negative effect of L_A on U_A^I

because the residents in city A can enjoy city A but also city B via the intercity transportation. As shown by Proposition 1, these two opposite effects interact to generate an optimal population size when the two cities are sufficiently complementary (such that $\varepsilon < 2$).

What is the role of complementarity? In general, when cities are more complementary, people are more likely to prefer to consume in both cities with a good balance. Due to this property of complementarity, when L_A is initially small, the positive effect of increasing L_A and thus $\alpha(L_A)$ dominates the negative effect of decreasing L_B, and thus $\alpha(L_B)$ is smaller because the intercity consumption is more balanced. This is why U_A^I increases with L_A if L_A is relatively small (more correctly, smaller than L_A^*). When L_A becomes sufficiently large, a further increase in L_A (and $\alpha(L_A)$) creates a bias in terms of the consumption in city B. As a result, the balance will be violated, so the negative effect of decreasing L_B and $\alpha(L_B)$ will dominate for $L_A > L_A^*$. Thus, an inverted U-shaped effect of L_A on U_A^I emerges due to the complementary interdependence between the cities supported by the existence of intercity transportation.

In order to completely characterize the property of indirect utility in our model, we then suppose that the cities are relatively substitutable in order to satisfy $\varepsilon > 2$. In this case, people prefer to consumer the more attractive city on a large scale. Thus, the indirect utility becomes higher if either city becomes very large. Therefore, the relationship between L_A and U_A^I is U-shaped for $\varepsilon > 2$.[3]

In summary, the welfare of a city resident can be an inverted U-shape even when the agglomerative economies of scale (population size) are very large and they cannot be balanced by diseconomies of scale due to congestion. The inverted U-shape occurs if the interdependence between cities are sufficiently enough, or more precisely, sufficiently complementary. Intercity dependence is possible due to the existence of intercity transportation. An important implication of our result is that even in an aging society, there might be an optimal population size for each city. Interdependence creates a negative effect of a population size on the welfare of residents in a similar manner to the congestion externality.

4 Equilibrium City Population Size with Free Migration

In this section, we investigate the equilibrium population size for a city in a sufficiently depopulated region, where we assume free migration: L_A increases (L_B decreases) over time if and only if $U_A^I > U_B^I$ ($U_A^I < U_B^I$). If $U_A^I = U_B^I$ holds, then no migration occurs, so L_A and L_B are constant.

As shown by Proposition 1, the indirect utility U_i^I can be an inverted U-shaped function of L_i if ε is not very large. There are various possible cases for migration. For the sake of simplicity, we start by considering a numerical example. We plot a graph where the horizontal axis measures L_A and the vertical axis measures $U_A^I(L_A)$ and $U_B^I(N-L_A)$. We set the parameters to $\varepsilon = 0.5$; $p_A = p_B = 1$; $q_{AB} = q_{BA} = 0.2$; and $\lambda = 0.5$. In this case, a globally stable migration equilibrium uniquely exists where $L_A = L_B = 2/N$. Thus, under free migration, the region contains multiple cities in the long run even if the regional population size has been sufficiently depopulated and it is so small that the city residents always benefit from their city's population size (Assumption 1).

In the next proposition, we formally derive a condition that ensures the long-run equilibrium with multiple cities. In order to focus on the role of intercity dependence and transportation, we assume symmetric cities such that $m_A = m_B$ and $p_A = p_B$.

[3] This is straightforward to prove using (10).

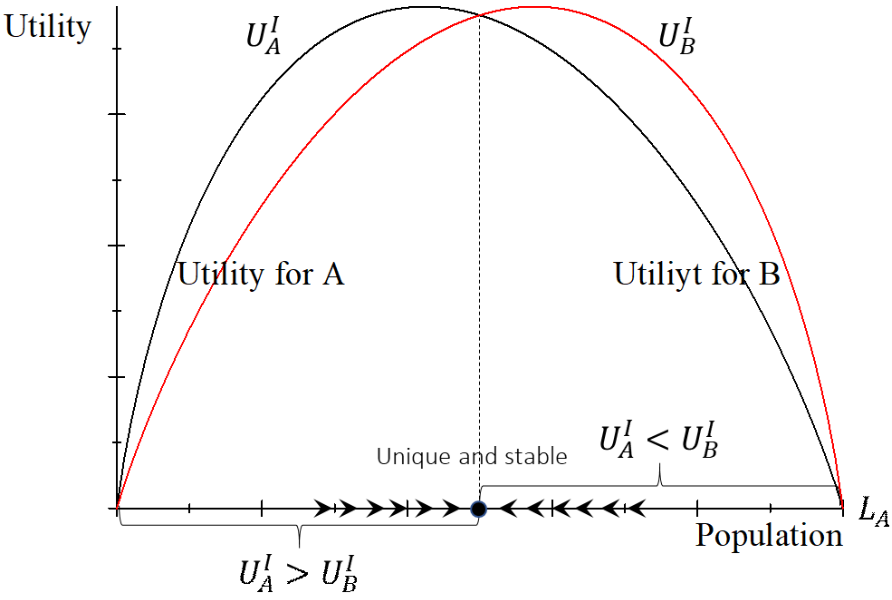

Figure 1: Figure 2: An example: Cities as gross complements

Proposition 2 *Under Assumption 1 regarding the depopulated region, free migration ensures that two cities can permanently coexist in the long run if and only if:*

$$\ln \lambda > \left(\frac{1-\varepsilon}{\varepsilon} \right) \ln q_{ij} \tag{11}$$

where $q_{ij} = q_{AB}$, q_{BA}. In this case, the steady-state population size for each city is given by:

$$L_i = \tilde{L}_i \equiv \frac{q_{ij} \left(\lambda^{\varepsilon} - (q_{ji})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}}{q_{AB} \left(\lambda^{\varepsilon} - (q_{BA})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} + q_{BA} \left(\lambda^{\varepsilon} - (q_{AB})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}} N \tag{12}$$

for $i = A$, B. Otherwise, a single-city equilibrium is realized in the long run, where $L_A \to N$ or $L_B \to N$ holds.

Proof. We define an intercity ration of indirect utility by:

$$\frac{U_A^I}{U_B^I} = \frac{m_A}{m_B} \left(\frac{\lambda^{\varepsilon} (p_A q_{AB})^{1-\varepsilon} (N - L_A)^{1-\varepsilon} + (q_{AB})^{1-\varepsilon} (p_B q_{BA})^{1-\varepsilon} (L_A)^{1-\varepsilon}}{(q_{BA})^{1-\varepsilon} (p_A q_{AB})^{1-\varepsilon} (N - L_A)^{1-\varepsilon} + \lambda^{\varepsilon} (p_B q_{BA})^{1-\varepsilon} (L_A)^{1-\varepsilon}} \right)^{\frac{1}{1-\varepsilon}} \equiv u(L_A). \tag{13}$$

We note that $u(L_A) \gtreqless (=) 1$ is equivalent to $U_A^I \gtreqless (=) U_B^I$. Differentiating $u(L_A)$ yields:

$$\frac{du(L_A)}{dL_A} = - \left(\lambda^{2\varepsilon} - (q_{AB} q_{BA})^{1-\varepsilon} \right) \Phi, \tag{14}$$

where $\Phi > 0$ is a composite of several variables.[4] By (14), u is a monotonically decreasing (increasing) function, $u(L_A)' < 0$, if and only if:

$$\lambda^{2\varepsilon} > (<) \left(q_{AB} q_{BA} \right)^{1-\varepsilon}. \tag{15}$$

Due to the monotonicity of u, the solution to $u(L_A) = 1$ is necessarily unique if it exists. This

[4] The formal definition is given by:

$$\Phi \equiv \frac{m_A}{m_B} \frac{N (p_A p_B q_{AB} q_{BA})^{1-\varepsilon}}{(N - L_A)^{\varepsilon} (L_A)^{\varepsilon}} \frac{\left(\lambda^{\varepsilon} (p_A q_{AB})^{1-\varepsilon} (N - L_A)^{1-\varepsilon} + (q_{AB})^{1-\varepsilon} (p_B q_{BA})^{1-\varepsilon} (L_A)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon} - 1}}{\left((q_{BA})^{1-\varepsilon} (p_A q_{AB})^{1-\varepsilon} (N - L_A)^{1-\varepsilon} + \lambda^{\varepsilon} (p_B q_{BA})^{1-\varepsilon} (L_A)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon} + 1}},$$

which is always strictly positive.

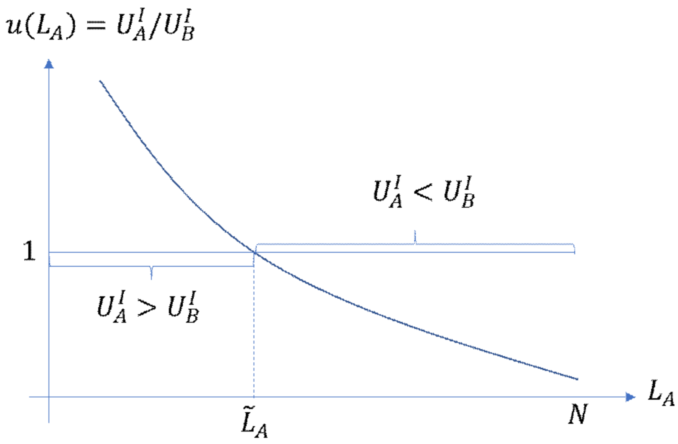

Figure 3: The intercity ratio of indirect utility

unique solution exists as a globally stable migration equilibrium if and only if $u(N) < 1 < u(0)$ holds,[5] as illustrated in Figure 3. It should be noted that:

$$u(0) = \begin{cases} \frac{m_A}{m_B} \frac{\lambda^{\frac{\varepsilon}{1-\varepsilon}}}{q_{BA}} & \text{for } \varepsilon < 1 \\ \frac{m_A}{m_B} q_{AB} \lambda^{\frac{\varepsilon}{\varepsilon-1}} & \text{for } \varepsilon > 1 \end{cases} \text{ and } u(N) = \begin{cases} \frac{m_A}{m_B} \frac{q_{AB}}{\lambda^{\frac{\varepsilon}{1-\varepsilon}}} & \text{for } \varepsilon < 1 \\ \frac{m_A}{m_B} \frac{1}{q_{BA}\lambda^{\frac{\varepsilon}{\varepsilon-1}}} & \text{for } \varepsilon > 1 \end{cases}.$$

Thus, $u(N) < 1 < u(0)$ holds if and only if:

$$\lambda > \max\left\{ \left(\frac{m_A q_{AB}}{m_B}\right)^{\frac{1-\varepsilon}{\varepsilon}}, \left(\frac{m_B q_{BA}}{m_A}\right)^{\frac{1-\varepsilon}{\varepsilon}} \right\}. \tag{16}$$

Applying $p_A = p_B$ and $m_A = m_B$ to (16) yields $\varepsilon \ln(\lambda q_{ij}) > \ln q_{ij}$, and thus we obtain (11). Finally, by solving $u' = 0$ in (14), we obtain:

$$L_i = \frac{p_i q_{ij} \left(\lambda^\varepsilon (m_i)^{1-\varepsilon} - (q_{ji} m_j)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}}{p_i q_{ij} \left(\lambda^\varepsilon (m_i)^{1-\varepsilon} - (q_{ji} m_j)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} + p_j q_{ji} \left(\lambda^\varepsilon (m_j)^{1-\varepsilon} - (q_{ij} m_i)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}} N. \tag{17}$$

By applying $p_A = p_B$ and $m_A = m_B$ again, (17) yields (12). ■

Proposition 2 implies an essential role for the interdependence between cities in the long-run existence of multiple cities, as well as for the existence of an optimal population size L_i^* (Proposition 1). In particular, three factors support this interdependence. The first factor is the stronger preference for outward consumption λ, which is intuitive because people with a higher λ are more willing to consume services in the other city. In this case, people favor the higher attractiveness of the other city, which strengthens the intercity dependence.

The second factor is the elasticity ε of substitution between cities ε. It is simple to verify that the right-hand side of (11) is increasing in $\varepsilon > 0$, and we note that $\ln q_{ij} \leq 0$ (due to $q_{ij} \leq 1$). Thus, for a smaller ε, the condition (11) is more likely to hold, which implies that the cities are more likely to coexist in the long run if they are more complementary. An important implication of this result is that the *complementary* interdependence between cities can be an important source of multiple cities.

The third factor is the quality of intercity transportation q_{ij}, which has more complex effects. The left-hand side of (11) is increasing in q_{ij} if the cities are gross complements with $\varepsilon < 1$,

[5] When $u' > 0$, we can easily verify that there is no stable equilibrium regardless of $u(L_A) = 1$ has a solution or not.

but decreasing if they are gross substitutes with $\varepsilon > 1$. Thus, for substitute cities, the quality q_{ij} of intercity transportation contributes to the long-run existence of multiple cities, which is a natural effect because transportation supports interdependence. However, for complementary cities, a higher quality q_{ij} of transportation *decreases* the likelihood of multiple-city equilibrium.

What is the mechanism responsible for this seemingly counterintuitive result? The mechanism is due to the standard property of gross complementarity because when cities are gross complements, a reduction in the price of city B's services leads to an increase in the consumption of city A's services. For residents in city A, a higher quality q_{AB} of transportation has a similar effect to a lower price in city B, so a higher q_{AB} leads to a larger consumption of their own city A. This increases the importance of their own city A's attractiveness, α_A, and its determinant, i.e., the population size L_A. Therefore, when the intercity transportation is higher quality, people need a larger population size for their own city rather than the other city's size. This is why the quality of intercity transportation q_{ij} damages the interdependence between complementary cities.

A more intuitive explanation is also possible. Thus, an increase in the quality of intercity transportation from city i to the other city, j, accompanied by a decrease in the cost for travelling consumption, leads to an increase in welfare (indirect utility) for the residents in city i, which then increases the population size L_i for city i in the multi-city equilibrium, as shown in (17). Therefore, effect of q_{ij} supports the multiple city equilibrium. However, if the quality q_{ij} is excessively high and it exceeds some cutoff level, then every individual in the region prefers to live in city i. In this case, in the long run equilibrium with free migration, the region converges to the situation where everyone moves to city i. As a result, city i is excessively attractive due to the good intercity transportation, and the other city j will cease to exist in the steady state equilibrium. This is another effect of q_{ij} and it is not conducive to the multiple city equilibrium.

The following remark summarizes the discussion given above.

Remark 1 *Three essential factors affect the interdependence between cities to determine whether multiple cities can persistently coexist in a depopulated region that is free of congestion.*

1. The preference for travelling consumption, λ, is always conducive to the persistence of multiple cities.

2. The complementarity between cities, $1/\varepsilon$, is always conducive to the persistence of multiple cities.

3. The quality of intercity transportation, q_{ij}, is conducive to the coexistence of substitute cities, but not conducive to the coexistence of complementary cities.

In particular, even if cities are gross substitutes ($\sigma > 1$), depopulation does not prevent the coexistence of multiple cities, when the preference for travelling consumption λ is sufficiently high and/or the quality of intercity transportation q_{ij} is sufficiently high. It should also be noted that our results do not depend essentially on the regional population size, N, which simply determines the optimal and equilibrium city population sizes in (8) and (12), but it is irrelevant to their existence. Thus, we can say that depopulation is a problem in the sense that the agglomerative economies of scale and diseconomies of scale due to congestion become less likely to be balanced, where continual depopulation can lead to regime switching in urban issues, as mentioned in the introduction. However, the problem might not be critical for the existence of optimal city population sizes and the sustainability of multiple cities. A possible implication of these findings is that regardless of how fast a population decreases or how small the total population might already be, city diversity can survive the era of depopulation due to the combined effect of the three forces stated in Remark 1.

Thus, we have considered a simple setting based on a city's attractiveness, α_i, where the attractiveness of city i, α_i, is a function of the population size in city i, L_i. However, in reality, various factors will affect α_i. One of the most important potential factors is the consumption externality, c_i. A synergy exists between different local and extraneous consumption types, c_i and c_{ij}. We consider that in extended settings, the attractiveness α_i itself could be an inverted U-shape as a function of the population size, L_i. These extensions are potentially interesting, but they will be investigated in future research.

In conclusion, the main finding obtained in this study is that *even in a simple setting where the city residents can always benefit from their city's population size (we modeled a sufficiently depopulated region that should typically emerge in the age of depopulation), the equilibrium welfare of the residents can be an inverted U-shape, and thus multiple cities can coexist permanently in equilibrium if the interdependence between cities is deeper due to the interactions of the three factors listed in Remark 1.*

5 Concluding Remarks

In this study, we considered the optimal/equilibrium city population size in a two-city model of endogenous migration. We assumed that the region is sufficiently depopulated so the agglomerative economies of population scale dominate diseconomies of scale, where the utility from living in a particular city increases monotonically with the city's population size on the feasible domain. We demonstrated that despite this monotonicity, the welfare of residents in a city can be an inverted U-shaped function of the city population if the interdependence between cities is more complementary, thereby suggesting that an optimal city population size might exist even when the regional population size has already become very small. We also showed that multiple cities can coexist permanently even in a sufficiently depopulated region if the interdependence between cities is sufficiently deep. Three factors support the depth of intercity dependence: (i) the preference for outward consumption, (ii) the complementarity between cities, and (iii) the quality of intercity transportation.

Among the various policy implications of our theoretical results, the following may be particularly interesting. When complementary cities persist in equilibrium, the quality of intercity transportation as a policy driver for civil engineering has an ambiguous role. Increasing the quality enhances the welfare of residents in both cities provided that it is within a moderate range. However, when the quality becomes excessively high and it exceeds the threshold level, all people try to move to only one of the two cities. Given that the welfare is an inverted U-shaped function of city population, the long run equilibrium with one large city is far from optimal. This suggests that a higher quality of intercity transportation is not always better for regional residents (in terms of welfare), depending on the degree of complementarity between cities. A moderate quality might be desirable for complementary cities in the era of depopulation.

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Appendix

In this appendix, we present an extended model that incorporates commuting for working, production, and general equilibrium. In the followings, we explain these three extensions and show that our results do not change in the extended model.

First, we introduce commuting in a simple manner where we assume that agents experience disutility from traveling due to the time used for commuting as well as for outside consumption. In particular, an agent who commutes from city i to j depreciates his/her utility by a rate of $\kappa_{ij} \in [0,1]$, thereby receiving $(1-\kappa_{ij})\,U_i \equiv \tilde{q}_{ij}U_i$. $\tilde{q}_{ij}$ denotes the quality of intercity transportation for commuting people. If $\tilde{q}_{ij} \neq q_{ij}$, then the quality of transportation differs depending on the purpose of travelling (consumption and commute) because of reasons such as the different schedules for trains/buses or different degrees of congestion. If $\tilde{q}_{ij} = q_{ij}$, then the quality is the same for consumption and commuting. We allow both possibilities, $\tilde{q}_{ij} \neq q_{ij}$ and $\tilde{q}_{ij} = q_{ij}$. Without any loss of generality, we assume that $\tilde{q}_{AB} \geq \tilde{q}_{BA}$.

We consider an equilibrium where some of the residents in city A commute to city B. In this case, $U_A^I = \tilde{q}_{AB}U_{AB}^I$ must hold, where U_{AB}^I denotes the indirect utility function for a resident of city A who commutes to city B and then earns m_B (rather than m_A). In an analogous manner to (4) and (5), from (1) and $m_B = p_Ac_A + p_Bc_{AB}$, we derive the consumptions for commuting residents in city A as:

$$c_A' = \frac{(p_A/\alpha_A)^{1-\varepsilon}}{(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon\,(p_B/(q_{AB}\alpha_B))^{1-\varepsilon}}\frac{m_B}{p_A} \text{ and } c_{AB}' = \frac{\lambda^\varepsilon\,(p_B/(q_{AB}\alpha_B))^{1-\varepsilon}}{(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon\,(p_B/(q_{AB}\alpha_B))^{1-\varepsilon}}\frac{m_B}{p_B}, \tag{A1a}$$

and the indirect utility function as:

$$U_{AB}^I = \frac{m_B}{\left[(p_A/\alpha_A)^{1-\varepsilon} + \lambda^\varepsilon\,(p_B/(q_{AB}\alpha_B))^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}. \tag{A1b}$$

By (5) and (A1b), $U_A^I = \tilde{q}_{AB}U_{AB}^I$ implies that $m_A = \tilde{q}_{AB}m_B$. With $\tilde{q}_{AB} \geq \tilde{q}_{BA}$, this implies that $m_B \geq q_{BA}m_A$, where no resident in city B commutes in equilibrium unless $\tilde{q}_{AB} = \tilde{q}_{BA} = 1$. We exclude $\tilde{q}_{AB} = \tilde{q}_{BA} = 1$ because this is a relatively trivial case.[6]

Next, we introduce production in a simple manner. We assume a linear production function as:

$$y_i = A\tilde{L}_i \quad \text{with } i = A,\ B, \tag{A2}$$

where y_i denotes the total output of city i's service, A denotes a parameter that captures the labor productivity in this region, and $\tilde{L}_i$ denotes the labor input for the services of city i. Perfectly competitive firms set the price, p_i, at a marginal cost level, w_i/A. Then, we have $w_i = p_iA$. Given that there is no other source of income in the model, $w_i = m_i$ holds. Thus, the non-arbitrage condition for commuting, $U_A^I = \tilde{q}_{AB}U_{AB}^I$, determines the relative price for the present case (with commuting from A to B) as:

$$\frac{p_A}{p_B} = \frac{m_A}{m_B} = \tilde{q}_{AB} \leq 1, \tag{A3}$$

noting $m_A = \tilde{q}_{AB}m_B$ and $m_i = p_iA$.

Then, we introduce the general equilibrium. There are two markets in the model of labor and city services. Market clearing for labor requires:

$$\tilde{L}_A = (1-\psi_{AB})\,L_A \text{ and } \tilde{L}_B = L_B + \psi_{AB}L_A, \tag{A4}$$

[6] It is easy to show that the situation where residents in city B commute to city A is impossible provided that $\tilde{q}_{AB} \geq \tilde{q}_{BA}$.

where $\psi_{AB} \in [0,1]$ denotes the fraction of residents in city A who commute to city B for work. Market clearing for city services requires:

$$c_A (1 - \psi_{AB}) L_A + c'_A \psi_{AB} L_A + c_{BA} L_B = y_A \text{ and } c_B L_B + c_{AB} (1 - \psi_{AB}) L_A + c'_{AB} \psi_{AB} L_A = y_B. \tag{A5}$$

When either of these two conditions holds, it is easy to verify that the other also holds, i.e., Walras's law holds.

In general equilibrium, conditions (A2), (A4), and (A5) determine the commuting fraction, ψ_{AB}, as a function of L_A/L_B. In addition, (A3) determines the relative price and income between city A and city B, which is equal to the quality of intercity transportation, $\tilde{q}_{AB}$. Finally, as shown in the main text, the non-arbitrage condition for migration determines the city population sizes, L_A and L_B, by comparing $U_A^I = \tilde{q}_{AB} U_{AB}^I$ and U_B^I.

We can show that the present extension to a general equilibrium model with commuting and production does not essentially change the analysis and the results given in the main text because the relative price and income are simply determined by $\tilde{q}_{AB}$ and independent of L_A and L_B. This simple finding is due to the allowance for commuting (and thus the non-arbitrage condition for commuting). We consider that from a realistic viewpoint, this allowance is reasonable because the region treated in the present study is not excessively large and some people typically commute between cities in reality.