Inventing Around, Trade in Similar Products, and Optimal Patent Breadths

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Abstract

This paper examines the effect of trade in imitated products, which are defined as products invented around, on innovators' behavior and patent breadths set by governments of trading countries. In particular, we compare unilateral patent breadth with the global optimum. We demonstrate that given patent breadths of trading countries, innovators' investment in invention of a new good is greater in an open economy than in a closed economy. We also find that when both home and foreign countries are symmetric on innovators' invention probability and imitators' entry probabilities, unilateral patent breadth of each government is narrower, which means that patent protection is laxer, than the global optimum. However, when the foreign country has strong comparative advantage in producing more similar products as compared with producing less similar products, unilateral patent breadth may be wider than the global optimum. We also refer to common patent breadth and the relationship between patent length and breadth.

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1. Introduction

For the past several decades, the intellectual property rights (IPR) have been strengthened in many countries. In particular, developing countries have marched with the trend of enforcing strict IPRs since the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS agreement) became effective in 1995. The fixed term length of patents has been harmonized internationally, and for many countries including developing countries, the patent length is 20 years that is applied to all categories of products across different industries. Some countries and/or negotiation on economic partnership agreement (EPA) are trying to extend the term.

Some say that this term is too long because there is trade-off relation between incentivizing development of new products and encouraging diffusion of those products. Longer patent protection gives stronger incentives to innovators to develop new products by ensuring larger monopoly profits, while entry of other firms that can supply those new products is delayed and, accordingly, higher prices of new products last longer.

On the other hand, others say that the long patent protection may be meaningless for certain industries because similar products are invented around the original patent and it is usually the case that the similar products (hereinafter, *imitation* throughout the paper) can be successfully supplied before the original patent expires. When an industry faces fierce competition, the actual term of patent protection can be much shorter than the predetermined fixed term by the law. For example, in the market for Liquid Crystal Display (LCD) panel, SHARP is the original patent holder of many technologies relating to LCD and was a dominant manufacturer up to the fourth generation of the panel toward the end of 20th century. However, the dominance did not last for the full length period of their patents. Taiwanese and Korean manufacturers succeeded to produce imitations, and in less than 10 years, the market share of Japanese manufactures drastically dropped within 10 years from 1997 through 2006.

The story of SHARP implies that not only the length but also the breadth of patent protection is important. As compared with the length of patent protection, the rules on the breadth are relatively ambiguous in the TRIPS agreement. For example, novelty is needed for a product or production method to be patented. However, the judgment on novelty is partly left to discretion of the authority of each country.[1] Recently, the Korean Intellectual Property Office (KIPO) judged that POSCO, the Korean steel maker, did not violate the patents of steel-processing technologies, called grain oriented electrical steel, which Nippon Steel & Sumitomo Metal (NSSM) claims exclusively belonged to it. The point of the judgment was novelty: KIPO judged that the technologies of NSSM are not new but similar to existing technologies. As a result, POSCO is able to keep supplying this type of steel, and the market share of NSSM becomes smaller than when those patents are approved.[2]

Authorities of parties of TRIPS agreement are also able to issue compulsory licensing if it contributes to public welfare. In particular, this rule has been drawing attention in relation to diffusion of new drugs. India is famous for generic drug production. In some cases, the Indian authority issue compulsory licensing by which domestic drug makers can supply new drugs patented by innovators in developed countries. For example, the government of India issued compulsory licenses for cancer drugs.[3] Compulsory licenses have been granted not only to the pharmaceutical industry but also other industries across the world.

According to Surveys on R&D activities by private companies 2014 conducted by National Institute of Science and Technology Policy (NISTP), most firms answered that they were able to monopolize the market of products they invented less than three years.[4]

It is conspicuous that trade in imitations has been increasing worldwide. Some developed

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1 Novelty can be related both patent height and breadth. Beschorner (2008) described that patent height refers to vertical differentiation or quality improvements while patent breadth determines the degree of horizontal differentiation. The latter is also called patent scope. Beschorner (2008) also described that novelty is related to vertical differentiation. Although this paper focuses on horizontal differentiation, we consider that novelty is also related to horizontal differentiation.

2 For example, see Kyong-ae (2014) for this issue.

3 See, for example, Rupali (2014) and Seth (2014) for this issue.

4 See the website of NISTP (http://www.nistep.go.jp/en/). However, this survey is publicized only in Japanese.

countries have been encouraging the export of imitations, while imitators in developing countries have become competitive for the past few decades.[5] In some industries such as the steel industry, one firm is not always an innovator: which one is innovator depends on the product or production method. This trend implies that innovators can also exist in both developed and developing countries. It is natural to consider that the increase in trade of imitations affects patent strategies of innovators and patent breadth policy of authorities. Thus, it is important to consider patent breadth policies in the presence of trade in both innovated and imitated products.

There are two main purposes of this paper. The first one is to examine the effect of trade in imitations on behavior of innovators and patent breadth of trading countries. The second one is to compare unilateral optimal breadth (Non-cooperative equilibrium) with the optimal breadth in terms of global welfare (Cooperative equilibrium). Our analysis also suggests one of reasons why invented around takes place frequently and imitated products are supplied in the era of strengthened patent protection across the world.

Many articles have been studying innovation and imitation (see Benoit (1985), Pepall and Richards (1994), Green and Scotchmer (1995), Bessen and Maskin (2009), and Lnaginier (2011) among others). Our study focuses on the patent policy settings of trading countries. For a closed economy, Klemperer (1990), Richard and Shapiro (1990), Van Dijk (1995), Denicolo (1996, 1999), Matutes et al. (1996), O'Donoghue et al. (1998), Takalo (1998), Denicolo and Zanchetting (2002), Beschorner (2008), and Yiannaka (2009) investigated the optimal patent length, breadth, or/and height. Some articles considered that the number of imitators reflects patent breadth while others adopted Hoteling type of product differentiation model and

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5 See Peter (2008), Khomba (2012) for the case of generic drugs. See also the following articles. The article of *Business Wire* titled "Research and markets: profound research and investment prospect of China's generic drug market, 2012-2016," on September 6th, 2012. The article of *Business Wire* titled "Emerging generic drug markets in Europe is a comparative study of high growth markets of opportunity," on December 19th, 2007.

considered that the minimum distance between innovated and imitated products is patent breadth.

Moreover, for open economies, Deardorff (1992), Zigic (1998), Grossman and Lai (2004), and Ivus (2011) tackled the patent policies for trading countries. However, they did not provide the behavior of imitators explicitly.

Our process of analysis is close to Grossman and Lai (2004): we determine the number of imitators endogenously. In terms of imitation cost, our analysis is closely related to Gallini (1992), which considered the combination of optimal patent length and breadth in the presence of costly imitation. However, she did not consider product differentiation explicitly. We extend Gallini (1992) and allow product differentiation, by which entry structure of imitators is endogenously determined and the effect of imitators' entry on patent breadth policies can be clearly examined.

Moreover, in terms of entry of imitators in an open economy, our analysis is based on Wright (2005) who investigated the optimal patent policies for trading countries, although, like Gallini (1992), he did not consider product differentiation explicitly. We extend Wright (2005) in the following three factors.

First, we adopt a Salop circle model to describe the difference/distance between innovated and imitated products.[6] Salop types of circle models have been adopted for analyzing firm behavior in the presence of product differentiation (Liu and Serfes, 2005, Geisler and Wiese, 2006, Brito and Pereira, 2010) and for investigating international trade or foreign outsourcing under imperfect competition (Helpman, 1981, Creane, 198, Yu, 2012). Eswaran and Gallini (1996) and Norman et al. (2016) adopted circle models to reflect patent breadth on the distance between products and examined optimal patent breadths. The former examined the relationship between optimal patent breadth on both product and process innovations and R&D costs and

[6] The seminal paper is Salop (1979), which investigated equilibria with product differentiation.

the latter investigated optimal patent breadth when subsequent innovation is built upon an already patented technology.

Second, we introduce innovators in both home and foreign countries to reflect the real situation. Wright (2005) distinguish a country with an innovator and imitators and a country only with imitators.

Third, Wright (2005) assumed that imitators enter only domestic markets, which implies that there is no trade in imitated products. However, imitators also enter foreign markets in the real world. This trend suggests the importance of considering entry determination of imitators and trade in their products, which we take into consideration in this paper.

We demonstrate that given patent breadths of trading countries, innovators' investment in invention of a new good is greater in an open economy than in a closed economy. We also find that when both home and foreign countries are symmetric on innovators' invention probability and imitators' entry probabilities, unilateral patent breadth of each government is narrower, which means that patent protection is laxer, than the global optimum. However, when the foreign country has strong comparative advantage in producing more similar products as compared with producing less similar products, unilateral patent breadth may be wider than the global optimum.

The structure of the paper is as follows. Section 2 describes the basic model in a closed economy. Section 3 examines a symmetric case in which invented and imitated products are traded internationally, while Section 4 examines an asymmetric case. Section 5 and 6 provides extensions and concluding remarks, respectively.

2. The Model

We begin with a closed economy to describe the basic structure of our model. Consumers are uniformly located on a circle of the market of a new good, which we call it as good x, with perimeter and density equal to L and 1, respectively. Each consumer buys at most one unit of good x. Utility depends on the basic utility (v),[7] the price of good p, and the distance between the locations of a consumer and a firm, which is an innovator or an imitator. For example, consider a consumer who is located at θ. When she purchases and consumes one unit of good x produced by a firm located at 0, her utility is given by

$$u = v - p - \theta, \quad if \; 0 \leq \theta < \frac{L}{2}, \tag{1}$$

$$u = v - p - (L - \theta), \quad if \; \frac{L}{2} \leq \theta < L. \tag{2}$$

The distance is considered as the difference between a consumer's ideal quality (her location) and the true quality of goods (the firm's location). In terms of patent policy, the distance also represents patent breadth, because it reflects *similarity* of neighbor products.

When an innovator succeeds in inventing a new good, it is located at 0, and imitators choose other points on the circle when they enter the market. We consider the following four-stage game. In the first stage, the government determines patent breadth which is the minimum distance an imitator has to keep from any incumbent. In the second stage, an innovator determines its investment amount in inventing a new good. Only when the innovator succeeds in invention, the game proceeds to the next stage. Imitators enter the market in the third stage, and the innovator and imitators compete on price in the fourth stage. We solve the game by backward induction.[8]

Before moving on to the fourth stage, we describe patent breadth policy and entry determination of imitators. The government controls m that determines patent breadth as follows:

$$Minimum\ Interval > \frac{L}{2^{m+1}}, \quad m \geq 0, \tag{3}$$

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[7] This utility can be considered as reservation price. See Eswaran and Gallini (1996) and Norman et al. (2016) among others.

[8] It may be possible for the government to fine-tune patent breadth after an innovator succeeds in inventing a new product. In such as case, the innovator is the first mover, and the government is the second mover. In this paper, we assume that the government is the first mover and that it determines the general strictness of patent breadth that can be applied for various industries.

The larger (smaller) is m, the narrower (wider) is patent breadth, and the laxer (stricter) is patent protection. We refer to the reason why we do not consider the number of imitators directly as a policy variable and instead consider that the government determines m in Setion 3.

We assume that there are small differences in the speed of inventing around among imitators: The most efficient imitator enters and chooses its location first, and then, the second most efficient imitator enters and chooses its location, and so on. In this case, if an imitator chooses its location so that the distance from the nearest rival is $L/2^m$, no additional imitator cannot choose its location between the first two firms that have already entered. Because each new imitator follows this strategy, there are $2^m - 1$ imitators and an innovator in this market in equilibrium under patent breadth policy given by (3). In addition, they are located at even intervals. In the following analysis, let $\bar{n}$ $(= 2^m)$ denote total number of firms including an innovator.

In the fourth stage, given number of imitators and locations of both innovator and imitators, each firm determines its price to maximize its profit. Because, as noted above, an innovator and imitators are located equi-distantly, location points are represented as:

$$l_i = \frac{iL}{\bar{n}}, \quad i = 0, \cdots, n-1. \tag{4}$$

See Figure 1 for the location of an innovator and imitators. Each consumer buys either one of neighbor products. From (1), the marginal consumer between firm 0, the innovator, and firm 1, the imitator next to the innovator, is represented as

$$v - p_0 - \theta_{01} = v - p_1 - \left(\frac{L}{\bar{n}} - \theta_{01}\right) \iff \theta_{01} = \frac{p_1 - p_0 + L/\bar{n}}{2},$$

where p_i denotes the price set by firm i. Similarly, the marginal consumer between firm i and firm $i+1$ is given by

$$\theta_{i,i+1} = \frac{p_{i+1} - p_i + (2i+1)L/\bar{n}}{2} \tag{5}$$

Thus, the profit of each firm in each period after invention and imitation is

$$\pi_i = p_i \cdot \left(\theta_{i,i+1} - \theta_{i,i-1}\right) \tag{6}$$

Note that the profit above does not include invention, imitation, and entry costs, which are fixed costs. We assume that the marginal production costs are zero. We also assume that the sum of the imitation and entry costs is not very large so that the profit of each imitator is non-negative for any patent breadth set by the government.[9] The first-order conditions (FOCs) for an innovator and imitators are given by

$$\frac{\partial \pi_i}{\partial p_i} = 0 . \tag{7}$$

From (5) and (6), (7) can be rewritten as

$$p_i = \frac{p_{i+1} + p_{i-1} + 2L/\overline{n}}{4} .$$

Because the innovator and imitators are located with even intervals, they are symmetric in price setting in the fourth stage. Thus, we obtain the following equilibrium price, quantity, and profits in each period:

$$\hat{p}_i = \frac{L}{\overline{n}}, \quad \hat{q}_i = \frac{L}{\overline{n}}, \quad \hat{\pi}_i = \left(\frac{L}{\overline{n}}\right)^2 \tag{8}$$

The marginal consumer is

$$\hat{\theta}_{i,i+1} = \frac{(2i+1)L}{2\overline{n}} . \tag{9}$$

Thus, consumer surplus with n firms in each period before patent expires is given by

$$C\hat{S} = 2\overline{n}\int_0^{L/2\overline{n}} \left(v - L/\overline{n} - z\right)dz = vL - \frac{5L^2}{4\overline{n}} \tag{10}$$

In the third stage, observing patent breadth set by the government, imitators determine whether they enter the market. As already described, imitators are located equi-distantly, and the number of imitators is determined uniquely given patent breadth. In reality, it takes certain

9 As we will explain the situation after expiration of patents, when there is no patent breadth policy, the number of imitators is determined so that the long-term variable profit is equal to a fixed imitation cost including entry cost. Our assumption implies that patent breadth set by the government is wider than the distance between any two firms in free-entry equilibrium.

periods for imitators to invent around. In those periods, an innovator is able to monopolize its market, which is the source of a larger profit for the innovator than profits for imitators. However, if these monopoly periods are taken into consideration, the timing of expiration of the original invention is also different from that of imitations, and the model becomes complicated without any additional benefits. Therefore, we assume that invention and inventing around take place in the same period for simplicity.[10]

In the second stage, the innovator determines its investment amount to maximize its expected profits, which is given by

$$\Pi_0 = \alpha(C_I)\cdot\left\{\int_0^{\overline{T}}\left(\frac{L}{\overline{n}}\right)^2 e^{-rz}dz + C_E e^{-r\overline{T}}\right\} - C_I, \qquad \alpha' > 0,\ \alpha'' < 0,\ \alpha(0) = 0,\ 0 \le \alpha < 1, \tag{11}$$

where α, C_I, C_E, $\overline{T}$, r denote the probability of succeeding invention, investment in invention, entry cost after patent expires, patent length, and discount rate, respectively. One unit of investment in innovation incurs one unit of cost, and the patent length is exogenous. When the patent of innovator expires, the patents of imitators also expire. Thus, any other firms are able to enter the market. However, each new entrant has to pay a fixed cost that is equal to C_E. Thus, the number of entrants are determined so that the long term variable profit is equal to C_E. This entry structure implies that the innovator also gains C_E after its patent expires, which is represented by $C_E e^{-r\overline{T}}$ in (11).

The FOC is given by

$$\frac{\partial \Pi_0}{\partial C_I} = \alpha'(C_I)\cdot\left\{\int_0^{\overline{T}}\left(\frac{L}{\overline{n}}\right)^2 e^{-rz}dz + C_E e^{-r\overline{T}}\right\} - 1 = 0 \tag{12}$$

It is obvious that the smaller is the number of imitators, the larger is investment.

In the first stage, the government determines patent breadth and, accordingly, the number of imitators before the patent of innovator expires to maximize expected welfare. Welfare is

10 We follow Gallini (1992) and Wright (2005) on this assumption.

defined as the sum of the profit of innovator, the profits of imitators, and consumer surplus. The profits of imitators after invention are given by

$$(\overline{n}-1)\Pi_i=(\overline{n}-1)\cdot\left\{\int_0^{\overline{T}}\left(\frac{L}{\overline{n}}\right)^2 e^{-rz}dz-C_M+C_E e^{-r\overline{T}}\right\},\quad i=1,\cdots,\overline{n}-1,\tag{13}$$

where C_M denotes entry cost before patent expires, which can also be considered as the cost of inventing around. Consumer surplus after invention is given by

$$C\overline{S}=\int_0^{\overline{T}}C\hat{S}\,e^{-rz}dz+\int_{\overline{T}}^{\infty}C\widetilde{S}\,e^{-rz}dz,\tag{14}$$

where $C\widetilde{S}$ denotes consumer surplus in each period after patents expire. Thus, expected welfare is written as

$$W=\alpha(C_I)\cdot\left\{\int_0^{\overline{T}}\left(\frac{L}{\overline{n}}\right)^2 e^{-rz}dz+C_E e^{-r\overline{T}}+(\overline{n}-1)\Pi_i+C\overline{S}\right\}-C_I,\ \ i=1,\cdots,\overline{n}-1.\tag{15}$$

In the following analysis, we do not describe the profits of firms, consumer surplus, and welfare after patents expire, because patent breadth, which is our focus, does not influence them (after expiration). We only describe the profit of the innovator, the profits of imitators, and consumer surplus before patents expire explicitly.

3. Trade in Invented and Similar Products: A Symmetric Case

In this section, assuming two trading countries are symmetric, we first compare an innovator's behavior in a closed economy with that in an open economy. Then, we examine governments' policy determination, in particular, compare unilateral patent breadth, which maximizes expected welfare of one country, with global optimum, which maximizes expected global welfare. The model described in the previous section is extended to the case in which there are two symmetric countries, which we refer to as home (country h) and foreign (country f) countries. The meaning of symmetry in this paper is as follows: (i) There is one innovator in each country; (ii) there are latent imitators in both countries, (iii) the cost function of innovation is identical for both innovators; (iv) entry costs of imitators are identical for all home and foreign imitators.

The four-stage game for a two-country model is as follows. In the first stage, both governments determine patent breadths that can be enforced in their own countries simultaneously. For the following analysis, let m_j $(j = h, f)$ denote patent breadth of country j and $\overline{n}_j = 2^{m_j}$ denote the number of firms including both an innovator and imitators in the market of country j. In the second stage, the home and foreign innovators determine their investment amounts in inventing a new good simultaneously. Only when either one innovator succeeds in invention, the innovator that has succeeded in invention enters both the home and foreign markets and is located at 0 in each market. We exclude the possibility that both innovators succeed in inventing new products that can be differentiated with each other at the same time. Then, the game proceeds to the next stage. Imitators enter both the home and foreign markets in the third stage, and the innovator and imitators compete on price in each market in the fourth stage. The third and fourth stages are the same as the case of a closed economy

Under symmetry, we consider that for each imitator that enters the markets, the probability that it is a home firm is $1/2$. As assumed in the previous section, we assume that there are small differences in the speed of inventing around among imitators. The most efficient imitator enters and chooses its location first, and then, the second most efficient imitator enters and chooses its location, and so on. As described by (3), because we consider that the number of firms determined by patent breadth as 2^{m_j}, each imitator chooses the same location in both markets as far as the number of entrants is smaller than that regulated by the stricter patent breadth, which is equivalent to the smaller m_j. When the maximum number of firms enter the market in either country with wider patent breadth, additional imitators enter the market in the other country with narrower patent breadth. See Figure 2 for an example of locations of imitators in both countries.

We set up the following assumption on entry costs of imitators: each imitator pays the entry cost (C_M) when it enters each market, which implies each imitator incurs $2C_M$ when it enters both home and foreign markets. The background for this assumption is as follows. The number of possible imitators is greater than that of imitators they are actually able to enter at least one of both home and foreign markets. When a new product is invented by either home or foreign innovator, all of those potential imitators invest in inventing around and compete with each other for imitation. In this sense, the total cost of inventing around does not depend patent breadths. On the other hand, when part of them succeed in inventing around faster than rival imitators, they fine-tune their product for consumers in each market. They also have to pay marketing costs. Thus, each imitator incurs entry cost when entering each market.

3.1 Behavior of Innovators

Now we consider the behavior of innovators in the second stage. Each innovator determines its investment amount to maximize its profit:

$$\Pi_{j,0} = \beta_j\left(C_j, C_k\right)\cdot\left(R_h + R_f\right) - C_j, \quad j = h, f, \quad j \neq k, \tag{16}$$

where C_j denotes investment amount by the innovator of country j. R_j $\left(j = h, f\right)$ is the revenue the innovator gains in the market of country j before its patent expires, which is given by

$$R_j = \int_0^{\overline{T}} \left(\frac{L}{\overline{n}_j}\right)^2 e^{-rz} dz. \tag{17}$$

Moreover, the probability of succeeding in invention and entering the market is defined as follows:

$$\beta_j\left(C_j, C_k\right) = \alpha_j\left(C_j\right)\cdot\left(1 - \alpha_k\left(C_k\right)\right) + \frac{\alpha_j\left(C_j\right)\alpha_k\left(C_k\right)}{2}, \qquad j,k = h, f, \quad j \neq k. \tag{18}$$

The FOC for each innovator is

$$\frac{\partial \Pi_{j,0}}{\partial C_j} = \alpha'_j \cdot \left(1 - \frac{\alpha_k}{2}\right) \cdot \left(R_h + R_f\right) - 1 = 0 \tag{19}$$

The second partial derivatives are

$$\frac{\partial^2 \Pi_{j,0}}{\partial C_j^2} = \alpha''_j \cdot \left(1 - \frac{\alpha_k}{2}\right) \cdot \left(R_h + R_f\right) < 0, \quad \frac{\partial^2 \Pi_{j,0}}{\partial C_k \partial C_j} = -\frac{\alpha'_j \alpha'_k}{2} \cdot \left(R_h + R_f\right) < 0. \tag{20}$$

The latter inequality implies that investment amounts of both innovators are strategic substitutes. We assume that the following inequality holds.

Assumption 1. $-\alpha''_j(2 - \alpha_k) > \alpha'_j \alpha'_k$

Under this assumption, from (20), the second-order conditions (SOCs) and stability condition are satisfied. From (12), we rewrite the FOC excluding the profit after patent expiration in a closed economy:

$$\frac{\partial \Pi_{j,0}}{\partial C_j} = \alpha'_j \cdot R_j - 1 = 0\,. \tag{21}$$

As far as patent breadths are the same, R_j in a closed economy is the same as that in an open economy. Moreover, $1 - \alpha_j/2 > 1/2$ holds. Thus, when both home and foreign patent breadth are identical, which implies that $R_j = R_k \ (j, k = h, f, j \neq k)$, comparison of (19) and (21) reveals the following result.

Result 1. *Suppose that both home and foreign patent breadth are identical. Then, given patent breadths, a change from a closed to an open economy increases the investment amounts of innovators.*

Intuition is as follows. An innovator is able to enter both home and foreign markets when it succeeds in inventing a new product in an open economy. The profits in an open economy is twice as much as those in a closed economy. Thus, in this respect, each innovator has stronger incentive to invest in innovation in an open economy than in a closed economy. On the other hand, given investment amount, each innovator has a smaller chance of inventing a new product in an open economy than in a closed economy, because there is a rival innovator in the former case while there is no rival in the latter case. In this respect, each innovator has weaker incentive to invest in innovation in an open economy than in a closed economy. The result reveals that the former positive effect on innovators' incentives dominates the latter negative effect.

Because $\partial\beta_j/\partial C_k < 0 \left(j,k = h, f,\ j \neq k\right)$, it holds that

$$\frac{\partial \Pi_{j,0}}{\partial C_k} < 0, \quad j,k = h, f, \quad j \neq k \tag{22}$$

which implies that investment amounts are excessive in terms of joint profit maximization of both innovators.

In a closed economy, a change in investment of an innovator in response to a change in patent breadth is obtained from (21) that

$$\frac{dC_j}{dm_j} = -\frac{\alpha'_j}{\alpha''_j R_j} \cdot \frac{dR_j}{d\overline{n}_j} \cdot 2^{m_j} \ln m_j < 0, \tag{23}$$

where $dR_j/d\overline{n}_j = -2\mu L^2 \overline{T}/\overline{n}_j^3 < 0$ and $\mu = \int_0^{\overline{T}} e^{-rz} dz$. On the other hand, the same response in an open economy is obtained from (19) that

$$\frac{dC_j}{dm_j} = -\frac{\alpha'}{\alpha'' \cdot \left(1-\alpha/2\right) - \alpha'^2/2} \cdot \frac{1}{R_h + R_f} \cdot \frac{dR_j}{d\overline{n}_j} \cdot 2^{m_j} \ln m_j < 0. \tag{24}$$

In (24), we use the condition of symmetry, that is, $\alpha_j = \alpha_k = \alpha$ in equilibrium after deriving the second partial derivatives. Comparison of (23) and (24) reveals that, given patent breadths set by both home and foreign governments, a change in investment of the home (foreign) innovator in response to a change in home (foreign) patent breadth may be larger or smaller in an open economy than in a closed economy.

From Result 1, given patent breadth, the investment amount of each innovator has a stronger incentive in an open economy than in a closed economy. In this respect, each innovator's response to a change in patent breadth is greater in an open economy than in a closed economy. However, in an open economy, a rival innovator also increases its investment amount when patent breadth becomes wider. Thus, as compared with a closed economy in the absence of rivals, an increase in the chance of inventing a new product is smaller in an open economy with a rival firm. In this respect, each innovator's response to a change in patent breadth is smaller in an open economy than in a closed economy. Which effect is greater depends on the shape of the function of innovation ($\alpha_j(C_j)$).

One point should be noted on the effect of a change in patent breadth of one country. We assume that both countries are symmetric and each innovator is able to enter both markets when succeeding in invention of a new product. Thus, a change in patent breadth of either one country has the same effect on both the home and foreign innovators as far as $m_j = m_k$, that is, $dC_j / dm_j = dC_j / dm_k$.

3.2 Patent Breadths

Now let us examine patent breadths set by governments. First, we briefly compare patent breadth in a closed economy with that in an open economy. Recalling that the probability that each imitator is a home (foreign) one is $1/2$, given patent breadths set by both home and foreign governments, the sum of the profits of home (foreign) imitators gained in the home (foreign) market and home (foreign) consumer surplus in an open economy are the same as those in a closed economy. The effects of a small change in patent breadth on those profits and consumer surplus in both cases are also equivalent. Result 1 remarks that, given patent breadth, innovators have stronger incentives to invest in innovation in the former case than in the latter case as far as both home and foreign patent breadths are equivalent. Thus, investment amounts of innovators are greater in the former case than in the latter case, which implies that governments are able to achieve a certain amount of investment by innovators with a narrower patent breadth or with weaker patent protection. Consequently, patent breadths are likely to be narrower in an open economy than in a closed economy.

Now let us compare unilateral patent breadth with global optimum. Expected welfare of country j is written as

$$W_j = \beta_j \cdot \left(R_h + R_f\right) - C_j + \gamma S_j, \qquad j = h, f \tag{25}$$

where

$$\gamma = \alpha_h + \alpha_f - \alpha_h \alpha_f, \tag{26}$$

$$S_j = \frac{\left(\overline{n}_h - 1\right)R_h}{2} + \frac{\left(\overline{n}_f - 1\right)R_f}{2} - \frac{\left(\overline{n}_h + \overline{n}_f - 2\right)C_M}{2} + \int_0^{\overline{T}} \left(vL - \frac{5L^2}{4\overline{n}_j}\right) e^{-rz} dz . \tag{27}$$

We proceed step by step to obtain the result on the comparison between unilateral patent breadth and global optimum. We first assume the following inequalities.

Assumption 2. $\dfrac{d^2 C_j}{dm_j^2} < 0, \ \dfrac{d^2 C_j}{dm_k^2} < 0, \ \dfrac{d^2 C_j}{dm_k dm_j} < 0.$

The intuition for the first two inequalities is as follows. When home (foreign) patent breadth is very narrow, or when home (foreign) patent protection is very weak, an increase in investment by the home (foreign) innovator in response to a stricter home (foreign) patent protection is large. However, as home (foreign) patent breadth becomes wider, investments amounts become larger, which implies that the effect of an additional increase in investment on the possibility of innovation becomes smaller. On the other hand, as home (foreign) patent breadth becomes wider, an increase in the profits of the home (foreign) innovator after innovation become larger. Those inequalities imply that the former negative effect dominates the latter positive effect. The last inequality has a similar meaning: the wider is the foreign (home) patent breadth, the smaller is a change in investment amount of the home (foreign) innovator due to a change in the home (foreign) patent breadth.

From the definition of R_j and the definition that $\overline{n}_j = 2^{m_j}$, it is obtained that

$$\frac{\partial R_j}{\partial m_j} = -2\mu L^2 \overline{T} / \overline{n}_j^3 \cdot 2^{m_j} \ln m_j < 0, \qquad \frac{\partial^2 R_j}{\partial m_k \partial m_j} = 0, \quad j,k = h, f, \quad j \neq k. \tag{28}$$

$$\frac{\partial^2 R_j}{\partial m_j^2} = \frac{2\mu L^2 \overline{T}}{\overline{n}^2} \cdot \left(2(\ln m_j)^2 - \frac{1}{m_j} \right) \tag{29}$$

If $m_j \geq 2$, which is equivalent to $\overline{n}_j \geq 4$, $2(\ln m_j)^2 > 1/m_j$ holds.[11] In this case, (29) is positive. Similarly, using $\overline{n}_j = 2^{m_j}$, we obtain

$$\frac{\partial S_j}{\partial m_j} = \frac{2^{m_j} \ln m_j}{2} \cdot \left(\frac{\mu L^2 \overline{T}}{2} \cdot \frac{3\overline{n}_j + 4}{\overline{n}_j^3} - C_M \right), \qquad \frac{\partial^2 S_j}{\partial m_k \partial m_j} = 0 \tag{30}$$

$$\frac{\partial^2 S_j}{\partial m_j^2} = -\frac{\mu L^2 \overline{T}}{4\overline{n}_j^2} \cdot \left\{ (3\overline{n}_j + 8) \cdot (\ln m_j)^2 - (3\overline{n}_j + 4) \cdot \frac{1}{m_j} \right\} - \frac{\overline{n}_j C_M}{2} \cdot \left((\ln m_j)^2 + \frac{1}{m_j} \right) \qquad 31)$$

Numeric calculation reveals that if $m_j \geq 2$, which is equivalent to $\overline{n}_j \geq 4$, the first braces of (31) is negative.[12] In this case, (31) is necessarily negative.

For the following analysis, we set up the following assumption.

Assumption 3. $m_j \geq 2$, $\dfrac{\partial S_j}{\partial m_j} > 0$.

S_j is the sum of imitators and consumer surplus of country j. Generally, stricter patent protection increases the expected profits of innovators, while it decreases consumer surplus and

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[11] If we allow that m_j is not an integer, it holds when $m_j \geq 1.72$.

[12] If we allow that m_j is not an integer, it holds when $m_j \geq 1.91$.

profits of imitators before patent expires. When considering this situation, the second inequality in Assumption 3 is intuitive. Unless C_M is large, this inequality is likely to hold theoretically.

Using the condition of symmetry of both home and foreign innovators, $C_h = C_f = C$ and $\alpha_h = \alpha_f = \alpha$ for any given combination of patent breadths, and the FOC for innovators, $\partial \Pi_{0,j} / \partial C_j = 0$, we obtain the FOC for the government of country j:

$$\frac{\partial W_j}{\partial m_j} = -\frac{\alpha\alpha'(R_h + R_f)}{2}\frac{dC}{dm_j} + \beta_j \frac{dR_j}{dm_j} + 2(1-\alpha)\alpha' S_j \frac{dC}{dm_j} + \gamma \frac{dS_j}{dm_j} = 0 \tag{32}$$

Under Assumptions 2 and 3, the SOCs are satisfied under a certain condition. See Appendix for the details. For the following analysis, we assume that the SOCs and stability condition are satisfied. Using $d\gamma/dm_j = (1-\alpha_k)\alpha_j' \, dC_j/dm_j + (1-\alpha_j)\alpha_k' \, dC_k/dm_j$, we also obtain that

$$\begin{aligned}\frac{\partial^2 W_j}{\partial m_k \partial m_j} = &-\frac{(\alpha'^2 + \alpha\alpha'')(R_h + R_f)}{2}\cdot\frac{dC}{dm_j}\frac{dC}{dm_k} - \frac{\alpha\alpha'}{2}\frac{dR_k}{dm_k}\frac{dC}{dm_j} - \frac{\alpha\alpha'(R_h + R_f)}{2}\frac{d^2C}{dm_k dm_j} \\ &+ (1-\alpha)\alpha'\frac{dC}{dm_k}\frac{dR_j}{dm_j} + 2((1-\alpha)\alpha'' - \alpha'^2)\alpha'\frac{dC}{dm_j}\frac{dC}{dm_k} + 2(1-\alpha)\alpha'\frac{dS_j}{dm_k}\frac{dC}{dm_j} \\ &+ 2(1-\alpha)\alpha' S_j \frac{d^2C}{dm_k\, dm_j} + 2(1-\alpha)\alpha'\frac{dS_j}{dm_j}\frac{dC}{dm_k}.\end{aligned} \tag{33}$$

From Assumptions 2 and 3, the second, fifth, sixth, seventh, and eights terms are negative. The third and fourth terms are positive, while the sign of the first term is negative. However, if

$$2\frac{dS_j}{dm_j} > -\frac{dR_j}{dm_j} \tag{34}$$

holds, the eighth term dominates the fourth term. In addition, if

$$4(1-\alpha)S_j > \alpha(R_h + R_f) \tag{35}$$

holds, the fifth term dominates the first term, and the seventh term dominates the third term. Thus, we obtain the following result.

Result 2. *Suppose that* $2\frac{dS_j}{dm_j} > -\frac{dR_j}{dm_j}$ *and* $4(1-\alpha)S_j > \alpha(R_h + R_f)$ *hold. Then, patent breadths set by both home and foreign governments are strategic substitutes.*

Let us consider the meaning of both inequalities in Result 2. The first inequality implies that the effect of a change in patent breadth of either country on the revenue of an innovator from the market of the country is smaller than the effect on the sum of profits of imitators and consumer surplus of the country. The comparison focuses on the periods after imitators enter the market. Thus, unless entry costs for imitators are very large, this inequality is likely to hold. Because imitators enter only when their expected profits are positive, this inequality also holds when there are sufficient number of consumers. The second inequality also holds as far as (i) entry costs for imitators are not very large and (ii) α is not very large.[13]

Now we focus on the factors that the home government does not take into consideration when determining its patent breadth unilaterally.[14] There are three factors: (i) the effect on the expected profit of the foreign innovator; (ii) the effect on the expected profits of foreign imitators; (iii) the effect on expected foreign consumer surplus. Although a change in home patent breadth does not affect the profits of the innovator and imitators gained from the foreign market and foreign consumer surplus in each period after invention, it influences them through a change in the probability of inventing a new product.

The effect of a change in home patent breadth on the expected profit of the foreign innovator is given by

$$\frac{d\Pi_{f,0}}{dm_h} = \beta_f \cdot \frac{\partial(R_h + R_f)}{\partial m_h} - \frac{\alpha_h \alpha_f'(R_h + R_f)}{2} \cdot \frac{dC_h}{dm_h} \tag{36}$$

Note that narrower patent breadth, which implies laxer patent protection and larger m_j, leads to larger number of imitators in the market of country j. The first term is negative which

[13] Since $0 \le \alpha < 1$, the fact that α is very large implies that it is close to one.

[14] Because symmetry is assumed, the foreign government's decision making can be examined in the same way.

implies that a narrower home patent breadth decreases the profit of the foreign innovator when it succeeds in inventing a new product. The second term is positive which implies that a narrower patent breadth decreases the rival innovator's investment by which the expected profit of the foreign innovator increases.

Because the probability that each imitator is a foreign one is $1/2$, the effect of a change in home patent breadth on the expected profits of foreign imitators gained in the home market is given by

$$\frac{1}{2}\cdot\frac{d\gamma(\overline{n}_h-1)(R_h-C_M)}{dm_h}=\frac{d\gamma}{dm_h}\cdot\frac{(\overline{n}_h-1)R_h}{2}+\frac{\gamma}{2}\cdot\frac{d(\overline{n}_h-1)R_h}{dm_h}-\frac{\gamma C_M}{2}\cdot\frac{d\overline{n}_h}{dm_h}. \tag{37}$$

The first term is negative which implies that a narrower home patent breadth decreases investments by innovators and, accordingly, decreases the probability of invention. The second term is also negative which implies that a narrower home patent breadth decreases the sum of the expected profits of foreign imitators because of fierce competition. This fact can be verified by using the definition of R_j. From (17), it is obtained that $(\overline{n}_h-1)R_h=\mu L^2(\overline{n}_h-1)/\overline{n}_h$, which gives rise to $d(\overline{n}_h-1)R_h/dm_h<0$. In addition, the third term is negative because a narrower home patent breadth increases the possible number of foreign imitators in the home market and, accordingly, total entry costs incurred by imitators increase.

The effect of a change in home patent breadth on the sum of expected foreign consumer surplus and expected profits of foreign imitators gained in the foreign market is given by

$$\frac{d\gamma\left(C\hat{S}_f+(\overline{n}_f-1)(R_f-C_M)/2\right)}{dm_h}=\frac{d\gamma}{dm_h}\cdot\left(C\hat{S}_f+\frac{(\overline{n}_f-1)(R_f-C_M)}{2}\right), \tag{38}$$

where

$$C\hat{S}_f=\int_0^{\overline{T}}\left(vL-\frac{5L^2}{4\overline{n}_f}\right)e^{-rz}dz.$$

(38) is negative which implies that a narrower home patent breadth decreases the probability of invention and, accordingly, decreases the sum of expected foreign consumer surplus and expected profits of imitators when a new product is invented.

Recalling that $d\gamma/dm_j = (1-\alpha_k)\alpha_j' \, dC_j/dm_j + (1-\alpha_j)\alpha_k' \, dC_k/dm_j$, if

$$2(1-\alpha)\{(\overline{n}_h - 1)R_h + (\overline{n}_f - 1)R_f\} > \alpha(R_h + R_f)$$

holds, the sum of the first term of (37) and the second term of (38) dominates the second term of (36). In this case, the home government ignores its negative effect of an increase in imitators in the home market on foreign welfare when it chooses patent breadth unilaterally. Thus, we obtain the following result.

Result 3. *If* $2(1-\alpha)\{(\overline{n}_h - 1)R_h + (\overline{n}_f - 1)R_f\} > \alpha(R_h + R_f)$ *holds, unilateral patent breadth of each government is narrower (unilateral patent protection is laxer) than the global optimum.*

Three points should be noted. First, $2(1-\alpha)\{(\overline{n}_h - 1)R_h + (\overline{n}_f - 1)R_f\} > \alpha(R_h + R_f)$ is a sufficient condition for Result 3 to hold, which is satisfied when the number of imitators is not very small or/and α is not very large. Even if this inequality is not satisfied, Result 3 may hold.

Second, in the introduction, we cited an example of steel industry. In the industry, there are innovators in several developed countries, and it is likely that one firm is not always an innovator. Focusing on those developed countries, the analysis of the symmetric case may be able to be applied to this type of industry.

Wright (2005) also obtained that unilateral patent protection is too weak in terms of global welfare. However, (36) and (37) are not taken into consideration in Wright (2005). We verify that even if (i) there are innovators in both countries, (ii) trade in imitations are taken into consideration, and (iii) patent breadths are focused on given patent length, the similar result is obtained.

4. Trade in Invented and Similar Products: An Asymmetric Case

In the real world, patent breadth may become an issue between developed and developing countries. In terms of innovation and imitation, developed and developing countries are not symmetric. In this section, we consider the following asymmetric case: (i) there is one innovator in the home country while there is no innovator in the foreign country; (ii) the probability that an imitator is a home firm is smaller than $1/2$. In particular, defining σ_i as the probability that the i-th efficient imitator is a home firm, we assume the following conditions.

Assumption 4. $\sigma_i < \frac{1}{2},\ \sigma_1 \geq \sigma_2 \geq \sigma_3 \cdots,\ \ i = 1,2,3,\cdots$ (39)

The first inequality implies that the foreign country has comparative advantage in inventing around/imitating as compared with innovative activities. The second inequalities imply that the foreign country has comparative advantage in producing more similar products as compared with producing less similar products. We consider that the foreign country has strong comparative advantage in producing more similar products when $\sigma_1 > \sigma_2 > \sigma_3 \cdots$.

In an open economy, the FOC for the home innovator is given by

$$\frac{\partial \Pi_{h,0}}{\partial C_h} = \alpha_h' \cdot \left(R_h + R_f\right) - 1 = 0 . \tag{40}$$

It is obvious that the SOC is satisfied. Thus, let us move onto the determination of patent breadths in an open economy. Expected home welfare is written as

$$W_h = \alpha_h \cdot \left(R_h + R_f\right) - C_h + \alpha_h S_h , \tag{41}$$

while expected foreign welfare is written as

$$W_f = \alpha_h S_f . \tag{42}$$

We also assume that the SOCs and stability condition are satisfied. Moreover, if Assumption 2 and two inequalities ((34) and (35)) hold, patent breadths set by both governments are strategic substitutes.

Comparison of unilateral patent breadth with the global optimum can be conducted in the same way as the symmetric case in the previous section. First, consider the case in which $\sigma_1 = \sigma_2 = \cdots \sigma_n = \tilde{\sigma}$.

Because there is no foreign innovator, there are two factors that the home government does not take into consideration when it determines its patent breadth unilaterally: (i) the effect of a change in home patent breadth on the expected profits of foreign imitators gained in the home market; (ii) the sum of expected foreign consumer surplus and expected profits of foreign imitators gained in the foreign market. In this case, (37) and (38) can be rewritten as

$$\begin{aligned}\frac{d\alpha_h(1-\tilde{\sigma})(\bar{n}_h-1)(R_h-C_M)}{dm_h} &= \frac{d\alpha_h}{dm_h}\cdot(1-\tilde{\sigma})\cdot(\bar{n}_h-1)(R_h-C_M)+\alpha_h\cdot(1-\tilde{\sigma})\cdot\frac{d(\bar{n}_h-1)R_h}{dm_h} \\ &\quad -\alpha_h(1-\tilde{\sigma})C_M\frac{d\bar{n}_h}{dm_h}\end{aligned} \tag{43}$$

$$\frac{d\alpha_h\left(C\hat{S}_f+(1-\tilde{\sigma})(\bar{n}_f-1)(R_f-C_M)\right)}{dm_h} = \frac{d\alpha_h}{dm_h}\cdot\left(C\hat{S}_f+(1-\tilde{\sigma})(\bar{n}_f-1)(R_f-C_M)\right) \tag{44}$$

On the other hand, the foreign government does not take into consideration the same three factors as the symmetric case. Thus, the same result is obtained on the comparison of unilateral patent breadth and the global optimum.

Result 4. *Suppose that* $\sigma_1 = \sigma_2 = \cdots \sigma_n < 1/2$. *Then, if* $2(1-\alpha)\{(\bar{n}_h-1)R_h+(\bar{n}_f-1)R_f\} > \alpha(R_h+R_f)$ *holds, unilateral patent breadth of each government is narrower (unilateral patent protection is laxer) than the global optimum.*

Second, we consider the case in which the foreign country has strong comparative advantage in producing more similar products, that is $\sigma_1 > \sigma_2 > \sigma_3 \cdots$. In this case, (43) and (44) can be rewritten as follows:

$$\begin{aligned}\frac{d\alpha_h(1-\breve{\sigma}_h)(\bar{n}_h-1)(R_h-C_M)}{dm_h} = &\frac{d\alpha_h}{dm_h}\cdot(1-\breve{\sigma}_h)\cdot(\bar{n}_h-1)(R_h-C_M)+\alpha_h\cdot(1-\breve{\sigma}_h)\cdot\frac{d(\bar{n}_h-1)R_h}{dm_h} \\ &-\frac{d\breve{\mu}_h}{dm_h}\cdot(\bar{n}_h-1)(R_h-C_M)-\alpha_h(1-\breve{\sigma}_h)C_M\frac{d\bar{n}_h}{dm_h}\end{aligned}, \quad (45)$$

$$\frac{d\alpha_h\left(C\hat{S}_f+(1-\breve{\sigma}_f)(\bar{n}_f-1)(R_f-C_M)\right)}{dm_h} = \frac{d\alpha_h}{dm_h}\cdot\left(C\hat{S}_f+(1-\breve{\sigma}_f)(\bar{n}_f-1)(R_f-C_M)\right), \quad (46)$$

where $\breve{\sigma}_j$ denotes the expected ratio of the number of home imitators to total number of imitators in the market of country j. Because $\sigma_1 > \sigma_2 > \sigma_3 \cdots$, $d\breve{\sigma}_h/dm_h < 0$ holds. Thus, additional effects, which are the third term of the right-hand side of (45), are positive. When the foreign country has comparative advantage in inventing around/imitating as compared with innovative activities and has strong comparative advantage in producing more similar products as compared with producing less similar products, this positive effect may dominate the other negative effects. In such a case, unilateral home patent breadth is wider, which implies that home patent protection is stricter, than the global optimum.

Regarding the factors that the foreign government does not take into consideration, this additional effect is negative, because $d\breve{\sigma}_f/dm_f > 0$. Thus, the same result as Results 3 and 4 is obtained for the foreign unilateral patent breadth.

Result 5. *Suppose that the foreign country has strong comparative advantage in producing more similar products. Then, unilateral patent breadth of the home country may be wider (unilateral patent protection may be stricter) than the global optimum.*

It should be noted that not only asymmetry on innovators but also asymmetry on imitators' ability are important factors for determining whether unilateral patent breadth is narrower than the global optimum. Contrast to heavy and chemical industries, in the case of many other industries such as pharmaceutical, clothing, and household appliance industries, invention and imitation structures are often asymmetric among countries, in particular, between developed and developing countries. In such a case, it may be that patent breadth of developed countries may be wider than the global optimum.

5. Extension

5.1 Common Patent Breadth

We have so far focused on unilateral patent breadth and global optimum. However, we often observe that some countries, in particular developed countries, advocate that patent protection should be harmonized and strengthened.[15] In this case, governments of those counties consider common patent policies among countries. Common patent breadth means that the same patent breadth is enforced in trading countries. As noted in the introduction, harmonization has been achieved regarding patent length, while it has not been achieved yet regarding patent breadth. In this subsection, we consider common patent breadth that is desirable for one country, home or foreign, and compare it with global optimal common patent breadth.

First, we consider a symmetric case as we focus on in Section 3. Different from unilateral patent breadth, a change in common patent breadth influences not only home consumer surplus and profits of firms gained in the home market but also foreign consumer surplus and profits of firms gained in the foreign market. Similar to the case of unilateral patent breadth, the home government does not take into consideration the effect on the expected profit of the foreign innovator, the effect on the expected profits of foreign imitators, and the effect on expected

[15] Wright (2005) also referred to this situation.

foreign consumer surplus when considering a desirable common patent breadth for the home country. However, under symmetry, those effects are the same as the effect on the expected profit of the home innovator, the effect on the expected profits of home imitators, and the effect on expected home consumer surplus, respectively, which are taken into consideration by the home government. Thus, it is clear that a common patent breadth desirable for one country is the same as the global optimal common patent breadth.

Second, we turn to an asymmetric case. Similar to the symmetric case, the home government takes into consideration three factors relating to home firms and consumers, while it does not take into consideration three factors relating to foreign firms and consumers. However, when imitators are asymmetric, the effect on the profits of home imitators is different from that on the profits of foreign imitators. For example, consider a case in which $\sigma_1 = \sigma_2 = \cdots \sigma_n < 1/2$ holds. When home patent breadth becomes wider, not only the sum of the profits of home imitators but also the sum of the profits of foreign imitators increases, because competition among imitators is mitigated. Under symmetry, the magnitude of a change in the profits of home imitators is the same as that of a change in the profits of foreign imitators. However, under the condition that $\sigma_1 = \sigma_2 = \cdots \sigma_n < 1/2$, the former is smaller than the latter. Then, a common patent breadth desirable for the home country is determined based on the smaller increase in the profits of imitators, which implies that this common patent breadth is too narrow in terms of the foreign country. Thus, in this situation, a common patent breadth desirable for the home country, which has comparative advantage in innovation, is narrower than the global optimal common patent breadth. On the other hand, a common patent breadth desirable for the foreign country, which has comparative advantage in imitation, is wider than the global optimal common patent breadth. Similar to the analysis of the previous section, when $\sigma_1 > \sigma_2 > \sigma_3 \cdots$ holds, the home country does not take into consideration the loss of foreign imitators due to wider common patent breadth. Thus, common patent breadth for the home country may be wider than the global optimum.

5.2 Complementarity between Length and Breadth

In the literature, in particular in theoretical analyses, the relationship between patent length and breadth has been examined. In many cases, both length and breadth are considered as substitutes. When patent is protected for long periods, a desirable breadth is relatively narrow in terms of expected welfare. Similarly, when patent is widely protected, a desirable length is relatively short in terms of expected welfare. As the second extension, we consider this relationship by taking into consideration the profits of imitators.

Recalling the FOC for the government of country j in the symmetric case ((32)), we examine the effect of a change in $\overline{T}$ on this FOC, which is given by

$$\frac{\partial^2 W_j}{\partial \overline{T} \partial m_j} = -\frac{\left(\alpha'^2 + \alpha\alpha''\right)\left(R_h + R_f\right)}{2}\frac{dC}{d\overline{T}}\frac{dC}{dm_j} - \frac{\alpha\alpha'}{2}\frac{\partial\left(R_h + R_f\right)}{\partial \overline{T}}\frac{dC}{dm_j} - \frac{\alpha\alpha'\left(R_h + R_f\right)}{2}\frac{d^2 C}{d\overline{T}dm_j}$$

$$+\frac{d\beta_j}{d\overline{T}}\frac{dR_j}{dm_j} + \beta_j\frac{d^2 R_j}{d\overline{T}dm_j}$$

$$-\left(\alpha'^2 - 2(1-\alpha)\alpha''\right)S_j\frac{dC}{d\overline{T}}\frac{dC}{dm_j} + 2(1-\alpha)\alpha'\frac{dS_j}{d\overline{T}}\frac{dC}{dm_j} + 2(1-\alpha)\alpha' S_j\frac{d^2 C}{d\overline{T}dm_j}$$

$$+\frac{d\gamma}{d\overline{T}}\frac{dS_j}{dm_j} + \gamma\frac{d^2 S_j}{d\overline{T}dm_j} \tag{47}$$

From the definition of R_j ((17)), the second term is positive. From Assumption 1, the sixth term is positive. From the definition of S_j ((27)), the seventh term is negative. However, longer patent protection sacrifices a greater consumer surplus after patent expires. Thus, this effect on consumer surplus can be considered as positive. In general, longer patent protection encourages innovative activities by innovators. Thus, it is likely that an increase in $\overline{T}$ increases the probability of succeeding in invention. Thus, the ninth term is positive. From (30), the tenth term is positive. On the other hand, from (28), the fourth and fifth terms are negative. Moreover, the sign of the first, third, and eighth terms are ambiguous.

Thus, in total, the sign of (47) is ambiguous in general, which implies that it is possible that patent length and breadth are not substitutes but complements. When patent protection becomes longer, given probability of invention, the loss of total profits of imitators due to narrower patent protection becomes greater. In addition, a decrease of investment in invention in response to a narrower patent protection may become greater. These factors give the government an incentive to adopt a wider patent breadth as patent length becomes longer. Therefore, if these factors dominate other factors that give the government an incentive to adopt a narrower patent breadth, patent length and breadth are complements in terms of expected welfare when considering unilateral patent protection. It should be noted that this result cannot be obtained when we do not consider the behavior and profits of imitators explicitly.

6. Conclusion

In this paper, we examined the effect of trade in imitated products on innovators' behavior and patent breadths set by governments of trading countries. In particular, we compared unilateral patent breadth with the global optimum.

First, we demonstrated that given patent breadths of trading countries, innovators' investment in invention of a new good is greater in an open economy than in a closed economy.

Second, we found that when both home and foreign countries are symmetric on innovators' invention probabilities and imitators' entry probabilities, unilateral patent breadth of each government is narrower, which means that patent protection is laxer, than the global optimum. The reason is that each government does not take into consideration the positive effects of unilateral wider patent breadth on foreign firms and consumers.

Third, we found that even under an asymmetric case, in which there is only a home innovator and entry probability of home imitators is smaller than that of foreign imitators, the results may be the same as the symmetric case. However, when the foreign country has strong comparative advantage in producing more similar products as compared with producing less similar products, unilateral home patent breadth may be wider than the global optimum.

We also refer to common patent breadth and complementarity of patent length and breadth. It is sometimes discretionary for authorities to determine whether a new product is truly new one or a similar product to a product that has already been supplied to consumers. That is why patent breadth is often controversial in reality. It is important for authorities to cooperate with each other to set clear patent breadths to balance promotion of invention and diffusion of new products. In such cases, not only asymmetry on innovators but also asymmetry on the ability of imitators are critical factors for determining desirable patent policies.

Appendix

The SOC for Governments

From (32), it is obtained that

$$\frac{\partial^2 W_j}{\partial m_k \partial m_j} = -\frac{(\alpha'^2 + \alpha\alpha'')(R_h + R_f)}{2} \cdot \left(\frac{dC}{dm_j}\right)^2 - \frac{\alpha\alpha'}{2}\frac{dR_j}{dm_j}\frac{dC}{dm_j} - \frac{\alpha\alpha'(R_h + R_f)}{2}\frac{d^2C}{dm_j^2} + \beta_j \frac{d^2R_j}{dm_j^2}$$

$$+ (1-\alpha)\alpha'\frac{dC}{dm_j}\frac{dR_j}{dm_j} + 2((1-\alpha)\alpha'' - \alpha'^2)S_j\left(\frac{dC}{dm_j}\right)^2 + 2(1-\alpha)\alpha'\frac{dS_j}{dm_j}\frac{dC}{dm_j}$$

$$+ 2(1-\alpha)\alpha' S_j \frac{d^2C}{dm_j^2} + 2(1-\alpha)\alpha'\frac{dS_j}{dm_j}\frac{dC}{dm_j} + \gamma\frac{d^2S_j}{dm_j^2} \tag{A.1}$$

From Assumptions 2 and3, the second, fifth, sixth, seventh, eighth, ninth, and tenth terms are negative. The third and fourth terms are positive and the sign of the first term is ambiguous. If

$$4(1-\alpha)S_j > \alpha(R_h + R_f) \tag{A.2}$$

holds, the eighth term dominates the third term, and the sixth term dominates the first term. Because $\gamma = 2\beta_j$, if

$$-2\frac{d^2 S_j}{dm_j^2} > \frac{d^2 R_j}{dm_j^2} \qquad \text{(A.3)}$$

holds, the tenth term dominates the fourth term. Thus, if both inequalities, (A.2) and (A.3), hold, the SOC for each government is satisfied.

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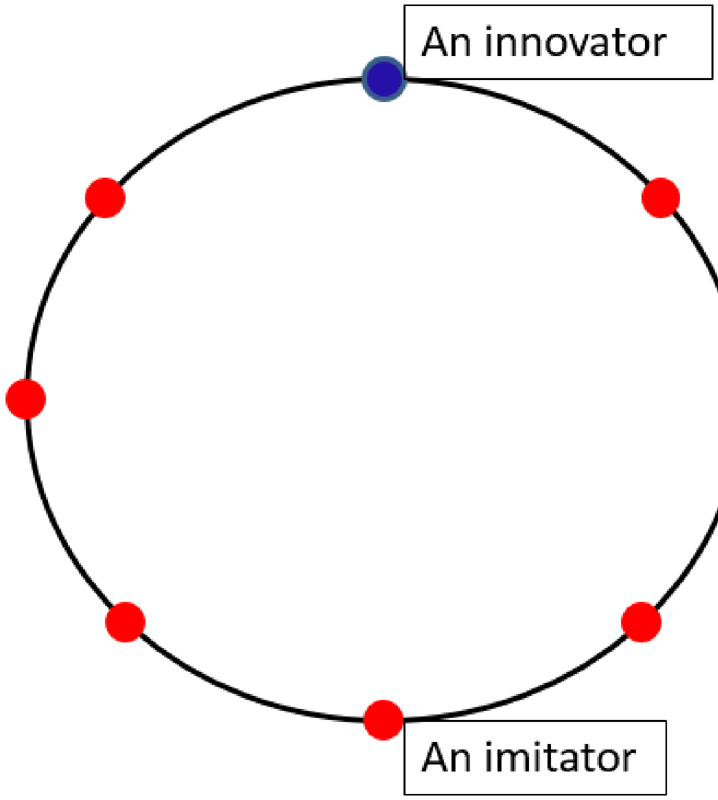

Figure 1. Location of an innovator and imitators

when m=3, that is, with 7 imitaotrs.

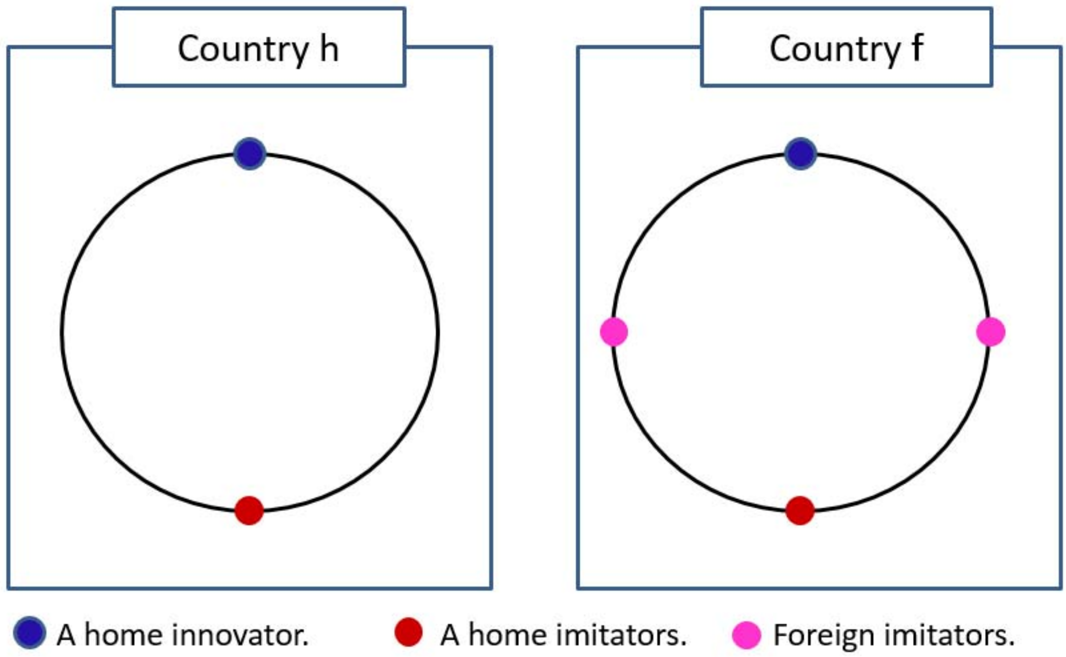

Figure 2. Location of an innovator and imitators

when mh=1 and mf=2 and the home innovator succeeds in invention.