Inventory holding and a mixed duopoly with a foreign joint-stock firm

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Abstract

This paper investigates a two-period mixed duopoly model in which a state-owned firm and a foreign joint-stock firm are allowed to hold inventories as a strategic device. The following situation is considered. In period one, each firm non-cooperatively decides how much it sells in the current market and the level of inventory it holds for the second-period market. By holding large inventories, a firm may be able to commit to large sales in period two. In period two, each firm non-cooperatively chooses its second-period production. At the end of period two, each firm sells its first-period inventory and its second-period production and holds no inventory. The paper traces out the reaction functions of the state-owned and foreign joint-stock firms in the mixed duopoly model with inventories.

Keywords: Inventory holding, state-owned firm, foreign joint-stock firm, reaction curves

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1. Introduction

Rotemberg and Saloner (1989) examine a two-period model in which inventories are used by a duopoly to deter deviations from an implicitly collusive arrangement, and establish that when demand is high, the incentive to deviate increases so that increases inventories may be optimal for the duopoly. Matsumura (1999) presents a Cournot duopoly model with finitely repeated competition, and establishes that two-period competition is insufficient to make private firms collusive. These studies investigate private duopoly models with inventories as a strategic device. Ohnishi (2011) presents a mixed market model in which a welfare-maximizing public firm and a profit-maximizing private firm can use inventory investment as a strategic device, and demonstrates that the equilibrium coincides with the Stackelberg solution where the private firm is the leader.

The analysis of mixed oligopoly models that incorporate state-owned public firms has been performed by many researchers (e.g., see Delbono and Rossini, 1992; Nett, 1994; Willner, 1994; Fjell and Pal, 1996; George and La Manna, 1996; White, 1996; Mujumdar and Pal, 1998; Pal, 1998; Pal and White, 1998; Poyago-Theotoky, 1998; Nishimori and Ogawa, 2002; Bárcena-Ruiz and Garzón, 2003; Ohnishi, 2006, 2009; Bárcena-Ruiz, 2007; Fernández-Ruiz, 2009; Heywood and Ye, 2010; Wang and Lee, 2010; Pal and Saha, 2014; Cracau, 2015). However, these studies consider mixed market models in which state-owned firms compete with capitalist or labor-managed firms, and do not include joint-stock firms.

Only few studies consider joint-stock firms. For example, Meade (1972) shows the differences in incentives, short-run adjustment and so forth among entrepreneurial, co-operative and joint-stock firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a homogeneous final good with inputs of capital and labor, and examines the behaviors of profit-maximizing, labor-managed and joint-stock firms. Ohnishi (2010b) presents the equilibrium solution of a quantity-setting model comprising a joint-stock firm and a profit-maximizing capitalist firm, and shows that introducing lifetime employment into the model of quantity-setting duopoly is beneficial only for the joint-stock firm. In addition, Ohnishi (2015b) investigates a mixed duopoly model where a joint-stock firm and a state-owned firm are allowed to offer lifetime employment as a strategic commitment, and presents the equilibrium solution of the mixed market model. However, these studies are mixed duopoly models with domestic joint-stock firms and do not include foreign joint-stock firms. To the best of my knowledge, there is no study that examines international mixed market models with state-owned and foreign joint-stock firms.

Therefore, we consider a two-period mixed market model in which a state-owned firm and a foreign labor-managed firm can hold inventories as a strategic device. The game runs as follows. In period one, each firm non-cooperatively decides how much it sells in the current market. In addition, each firm non-cooperatively decides the level of inventory it holds for the second-period market. We trace out the reaction functions of the state-owned and foreign joint-stock firms in the mixed duopoly model with inventories.

The balance of this paper is organized as follows. In the second section, we describe the model. The third section characterizes best replies for firms in the model. The fourth section presents the results of this study. The final section concludes the paper.

2. The model

We consider a mixed duopoly model with one domestic state-owned welfare-maximizing firm (firm D) and one foreign joint-stock profit-per-capital-maximizing firm (firm F), producing perfectly substitutable goods. In the balance of this paper, subscripts D and F refer to firms D and F, respectively, and superscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when i and j are used to refer to firms in an expression, they should be understood to refer to D and F with $i \neq j$. The price of each period is determined by $P(S^t)$, where $S^t = s_{\mathrm{D}}^t + s_{\mathrm{F}}^t$ is the aggregate sales of each period. We assume that $P' < 0$ and $P'' \leq 0$.

The two periods of the game run as follows. In period one, each firm non-cooperatively and simultaneously decides its first-period production $q_i^1 \in [0, \infty)$ and its first-period sales $s_i^1 \in [0, q_i^1]$. Firm i 's inventory I_i^1 becomes $q_i^1 - s_i^1$. In period two, each firm non-cooperatively and simultaneously decides its second-period production $q_i^2 \in [0, \infty)$. At the end of period two, each firm sells $s_i^2 = I_i^1 + q_i^2$ and holds no inventory. For notational simplicity, we consider the game without discounting.

Since $\sum_{t=1}^{2} q_i^t = \sum_{t=1}^{2} s_i^t$, economic welfare is

$$W=\sum_{t=1}^{2}\left[\int_0^{S^t}P(x)dx-m_{\mathrm{D}}q_{\mathrm{D}}^t-P(S^t)q_{\mathrm{F}}^t\right]=\sum_{t=1}^{2}\left[\int_0^{S^t}P(x)dx-m_{\mathrm{D}}s_{\mathrm{D}}^t-P(S^t)s_{\mathrm{F}}^t\right] \quad (1)$$

where $m_{\mathrm{D}}\in(0,\infty)$ denotes firm D's constant marginal cost. The demand and cost conditions that firms face remain unchanged over time. We define

$$w^t\equiv\int_0^{S^t}P(x)dx-m_{\mathrm{D}}s_{\mathrm{D}}^t-P(S^t)s_{\mathrm{F}}^t \quad (2)$$

In addition, since $\sum_{t=1}^{2}q_{\mathrm{F}}^t=\sum_{t=1}^{2}s_{\mathrm{F}}^t$, firm F's profit per capital is

$$\Phi_{\mathrm{F}}=\sum_{t=1}^{2}\left[\frac{P(S^t)s_{\mathrm{F}}^t-m_{\mathrm{F}}q_{\mathrm{F}}^t-f_{\mathrm{F}}^t}{k_{\mathrm{F}}(s_{\mathrm{F}}^t)}\right]=\sum_{t=1}^{2}\left[\frac{P(S^t)s_{\mathrm{F}}^t-m_{\mathrm{F}}s_{\mathrm{F}}^t-f_{\mathrm{F}}^{\mathrm{t}}}{k_{\mathrm{F}}(s_{\mathrm{F}}^t)}\right] \quad (3)$$

where $m_{\mathrm{F}}\in(0,\infty)$ denotes firm F's constant marginal cost, $f_{\mathrm{F}}\in(0,\infty)$ is firm F's fixed cost, and $k_{\mathrm{F}}(s_{\mathrm{F}}^t)$ is firm F's capital input function.

We assume that $k_{\mathrm{F}}(s_{\mathrm{F}}^t)$ is the function of s_{F}^t with $k_{\mathrm{F}}'>0$ and $k_{\mathrm{F}}''>0$. This assumption means that the marginal quantity of capital used is increasing.

We also assume that firm D is less efficient than firm F, i.e. $m_{\mathrm{D}}>m_{\mathrm{F}}$. This assumption is justified in Gunderson (1979) and Nett (1993, 1994), and is often used in literature studying mixed oligopoly markets (e.g., see George and La Manna, 1996; Mujumdar and Pal, 1998; Pal, 1998; Nishimori and Ogawa, 2002; Matsumura, 2003; Ohnishi, 2006, 2015a; Fernández-Ruiz, 2009). If firm D is equally or more efficient than firm F, then firm D chooses q_{D}^t and s_{D}^t such that price equals marginal cost. Therefore, firm F has no incentive to operate in the market, and firm D acts as a monopoly.

We define

$$\phi_{\mathrm{F}}^{t} \equiv \frac{P(S^{t})s_{\mathrm{F}}^{t} - m_{\mathrm{F}}s_{\mathrm{F}}^{t} - f_{\mathrm{F}}^{t}}{k_{\mathrm{F}}(s_{\mathrm{F}}^{t})} \tag{4}$$

The solution concept of this model is subgame perfection. In the next section, we will give supplementary explanations of the model.

3. Supplementary explanations

First, we derive firm D's reaction functions from (2). In period one, since there is no inventory available, firm D's reaction function is defined as:

$$R_{\mathrm{D}}^{1}(s_{\mathrm{D}}^{1}) = \arg\max_{\{s_{\mathrm{D}}^{1} \geq 0\}} \left[\int_{0}^{S^{1}} P(x)dx - m_{\mathrm{D}}s_{\mathrm{D}}^{1} - P(S^{1})s_{\mathrm{F}}^{1} \right] \tag{5}$$

In period two, firm D's reaction function without inventory is defined as:

$$R_{\mathrm{D}}^{2}(s_{\mathrm{D}}^{2}) = \arg\max_{\{s_{\mathrm{D}}^{2} \geq 0\}} \left[\int_{0}^{S^{2}} P(x)dx - m_{\mathrm{D}}s_{\mathrm{D}}^{2} - P(S^{2})s_{\mathrm{F}}^{2} \right] \tag{6}$$

and therefore its best response is shown as follows:

$$\overline{R}_{\mathrm{D}}^{2}(s_{\mathrm{F}}^{2}) = \begin{cases} R_{\mathrm{D}}^{2}(s_{\mathrm{F}}^{2}) & \text{if } s_{\mathrm{D}}^{2} > I_{\mathrm{D}}^{1} \\ I_{\mathrm{D}}^{1} & \text{if } s_{\mathrm{D}}^{2} = I_{\mathrm{D}}^{1} \end{cases} \tag{7}$$

Firm D maximizes economic welfare with respect to s_{D}^{t}, given s_{F}^{t}. The equilibrium solution needs to satisfy the following conditions: When the inventory is zero, the first-order condition for firm D is

$$P - m_{\mathrm{D}} - P's_{\mathrm{F}}^{t} = 0 \tag{8}$$

and the second-order condition is

$$P' - P''s_{\mathrm{F}}^{t} < 0 \tag{9}$$

Therefore, we obtain

$$R_{\mathrm{D}}^{t\,\prime}(s_{\mathrm{F}}^{t}) = \frac{P'' s_{\mathrm{F}}^{t}}{P' - P'' s_{\mathrm{F}}^{t}} \tag{10}$$

In period one, firm D's reaction function is upward sloping. In period two, firm D's best response also slopes upward for $s_{\mathrm{D}}^{2} > I_{\mathrm{D}}^{1}$. This means that firm D treats s_{D}^{t} as strategic complements.[1]

Next, we derive firm F's reaction functions from (4). In period one, since there is no inventory available, firm F's reaction function is defined as:

$$R_{\mathrm{F}}^{1}(s_{\mathrm{D}}^{1}) = \arg\max_{\{s_{\mathrm{F}}^{1} \geq 0\}} \left[\frac{P(S^{1}) s_{\mathrm{F}}^{1} - m_{\mathrm{F}} s_{\mathrm{F}}^{1} - f_{\mathrm{F}}^{1}}{k_{\mathrm{F}}(s_{\mathrm{F}}^{1})} \right] \tag{11}$$

In period two, firm F's reaction function without inventory is defined as:

$$R_{\mathrm{F}}^{2}(s_{\mathrm{D}}^{2}) = \arg\max_{\{s_{\mathrm{F}}^{2} \geq 0\}} \left[\frac{P(S^{2}) s_{\mathrm{F}}^{2} - m_{\mathrm{F}} s_{\mathrm{F}}^{2} - f_{\mathrm{F}}^{2}}{k_{\mathrm{F}}(s_{\mathrm{F}}^{2})} \right] \tag{12}$$

and therefore its best response is shown as follows:

$$\overline{R}_{\mathrm{F}}^{2}(s_{\mathrm{D}}^{2}) = \begin{cases} R_{\mathrm{F}}^{2}(s_{\mathrm{D}}^{2}) & \text{if } s_{\mathrm{F}}^{2} > I_{\mathrm{F}}^{1} \\ I_{\mathrm{F}}^{1} & \text{if } s_{\mathrm{F}}^{2} = I_{\mathrm{F}}^{1} \end{cases} \tag{13}$$

Firm F maximizes its profit per capital with respect to s_{F}^{t}, given s_{D}^{t}. When the inventory is zero, the first-order condition for firm F is

$$(P' s_{\mathrm{F}}^{t} + P - m_{\mathrm{F}}) k_{\mathrm{F}} - (P s_{\mathrm{F}}^{t} - m_{\mathrm{F}} s_{\mathrm{F}}^{t} - f_{\mathrm{F}}^{t}) k_{\mathrm{F}}' = 0 \tag{14}$$

and the second-order condition is

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[1] The concept of strategic complements is due to Bulow, Geanakoplos, and Klemperer (1985).

$$(P''s_{\mathrm{F}}^{t} + 2P')k_{\mathrm{F}} - (Ps_{\mathrm{F}}^{t} - m_{\mathrm{F}}s_{\mathrm{F}}^{t} - f_{\mathrm{F}}^{t})k_{\mathrm{F}}'' < 0 \tag{15}$$

In addition, we obtain

$$R_{\mathrm{F}}^{t}\,'(s_{\mathrm{D}}^{t}) = -\frac{P''s_{\mathrm{F}}^{t}k_{\mathrm{F}} + P'(k_{\mathrm{F}} - s_{\mathrm{F}}^{t}k_{\mathrm{F}}')}{(P''s_{\mathrm{F}}^{t} + 2P')k_{\mathrm{F}} - (Ps_{\mathrm{F}}^{t} - m_{\mathrm{F}}s_{\mathrm{F}}^{t} - f_{\mathrm{F}}^{t})k_{\mathrm{F}}''} \tag{16}$$

Since $k_{\mathrm{F}}'' > 0$, $k_{\mathrm{F}} - s_{\mathrm{F}}^{t}k_{\mathrm{F}}' < 0$, so that $P''s_{\mathrm{F}}^{t}k_{\mathrm{F}} + P'(k_{\mathrm{F}} - s_{\mathrm{F}}^{t}k_{\mathrm{F}}')$ is positive. We see that firm F treats s_{F}^{t} as strategic complements.

4. Results

In this section, we trace out the firms' reaction curves in the model described in section 2. There is no inventory available in period one, and further s_i^1 does not affect s_i^2 and s_j^2. Firm D's and firm F's reaction curves in period two is decided by the level of I_i^1, and therefore we consider period two.

We illustrate both firms' reaction curves by using Figures 1-12. For explanations, the figures are drawn simply. We discuss the following four cases.

Case 1: Neither firm holds inventory.

Case 2: Only firm D holds inventory.

Case 3: Only firm F holds inventory.

Case 4: Each firm holds inventory.

We discuss these cases in orders.

Case 1

Both firms' reaction curves are drawn in Figure 1, where R_i^2 denotes firm i 's second-period reaction curve with no inventory. Both firms' reaction curves are upward sloping. The equilibrium is decided in a Cournot fashion, i.e., the intersection of firm D's and firm F's second-period reaction curves gives us the equilibrium of the game. The reaction curves cross twice as depicted in Figure 1. Only point N is a stable Cournot equilibrium, since in point N' firm F's second-period reaction curve crosses firm D's from above. In the balance of this paper, we delete the superscript 2 for brevity's sake.

Case 2

This case is illustrated in Figures 2-4. We illustrate firm D's best response curve, which are drawn in Figure 2. We suppose that the firm D maintains the inventory level of I_{D}^{1B} in period two. By holding inventories, firm D's best response changes to (7). Firm D's inventory holding thus creates a kink in its reaction curve at the level of I_{D}^{1B}. Therefore, firm D's reaction curve becomes the kinked bold broken lines.

From Figure 2, we see that the inventory level of I_{D}^{1B} changes the solution of the game. The intersection of the reaction curves is the equilibrium sales in period two. That is, if firm

D holds I_{D}^{1B}, then the solution occurs at A. We see that economic welfare is higher at A than at N' and at N.

We consider the situation drawn in Figure 3. If firm D maintains the inventory level of I_{D}^{1C} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{D}^{1C}. The intersection of the reaction curves is the equilibrium sales in period two. That is, if firm D holds I_{D}^{1C}, then the solution becomes N'. It is easy to see that N' is an unstable equilibrium solution. If firm D maintains the inventory level of I_{D}^{1C}, then there is no stable solution.

We consider the situation drawn in Figure 4. If firm D holds I_{D}^{1E} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{D}^{1E}. Figure 4 shows that the intersection of the reaction curves is not affected by the kink. In this figure, the reaction curves cross at two points. We see that only N is a stable solution.

Case 3

This case is illustrated in Figures 5-7. Firstly, we consider the situation in Figure 5. We suppose that the firm F holds I_{F}^{1G} in period two. By holding inventories, firm F's best response changes to (13). Firm F's inventory holding thus creates a kink in its reaction curve at the level of I_{F}^{1G}. Therefore, firm F's reaction curve becomes the kinked bold lines.

From Figure 5, we see that the inventory level of I_{F}^{1G} changes the solution of the game. If firm F holds I_{F}^{1G}, then the solution is at F. However, we see that firm F's profit per capacity is lower at F than at N' and at N.

Secondly, we consider the situation drawn in Figure 6. If firm F maintains the inventory level of I_{F}^{1H} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{F}^{1H}. The intersection of the reaction curves is the equilibrium sales in period two. That is, if firm F maintains the inventory level of I_{F}^{1H}, then the solution becomes N'. In this figure, there is no stable solution.

Thirdly, we consider the situation in Figure 7. If firm F holds I_{F}^{1J} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{F}^{1J}. The best response curves cross at multiple points as in Figure 7.

Case 4

This case is illustrated in Figures 8-12. In this case, both firms use inventory holding as a strategic commitment device. Firstly, we consider the situation in Figure 8. Suppose that firm D maintains the inventory level of I_{D}^{1B} in period two. By holding inventories, firm D's best response changes to (7). Firm D's inventory holding thus creates a kink in its reaction curve at the level of I_{D}^{1B}. Therefore, firm D's reaction curve becomes the kinked bold broken lines. In addition, suppose that firm F holds I_{F}^{1G} in period two. By holding inventories, firm F's best response changes to (13). Firm F's inventory holding thus creates a kink in its reaction curve at the level of I_{F}^{1G}. Therefore, firm F's reaction curve becomes the kinked bold lines.

The solution is decided in a Cournot fashion. From Figure 8, we see that inventory holding by each firm changes the solution of the game. The intersection of new reaction curves is the equilibrium sales in period two. Figure 8 indicates that there are three stable solutions.

Secondly, we consider the situation in Figure 9. If firm D maintains the inventory level of I_{D}^{1C}, then its best response curve becomes the bold broken lines, and if firm F holds I_{F}^{1J}, then its best response curve becomes the bold lines. In Figure 9, both firms' reaction curves do not cross each other. That is, the obvious outcome is that there is no solution.

Thirdly, we consider the situation in Figure 10. When firm D holds I_{D}^{1E} in the second period, its quantity best response curve is kinked at the level of I_{D}^{1E}. In addition, inventory holding by firm F kinks its quantity best response curve. Therefore, firm D's best response is depicted as the thick broken lines, and firm F's best response curve is the thick lines. The firms' best response curves cross twice as in Figure 10. We see that both N and D are stable solutions.

Fourthly, we consider the situation drawn in Figure 11. Inventory holding by each firm kinks its quantity best response curve. If firms D and F hold I_{D}^{1W} and I_{F}^{1T} respectively, then firm D's best response is depicted as the thick broken lines, and firm F's best response curve is the thick lines. In this figure, the firms' best response curves cross at multiple points. We see that both N and M are stable solutions.

Fifthly, we consider the case drawn in Figure 12. We suppose that firms D and F hold I_{D}^{1W} and I_{F}^{1J}, respectively. Therefore, firm D's best response is depicted as the thick broken lines, and firm F's best response curve is the thick lines. The firms' best response curves cross four times as in Figure 12. We see that all these points are stable solutions.

5. Conclusion

We have considered a two-period mixed market model in which a state-owned firm and a foreign joint-stock firm are allowed to hold inventories as a strategic device. As a result, we have shown that there may be multiple stable Cournot solutions in the international mixed duopoly model.

In this paper, we have examined an international mixed duopoly model where a state-owned firm competes with a foreign joint-stock firm. In the near future, we will extend our analysis by considering a mixed oligopoly model where a state-owned firm competes with both domestic and foreign joint-stock firms.

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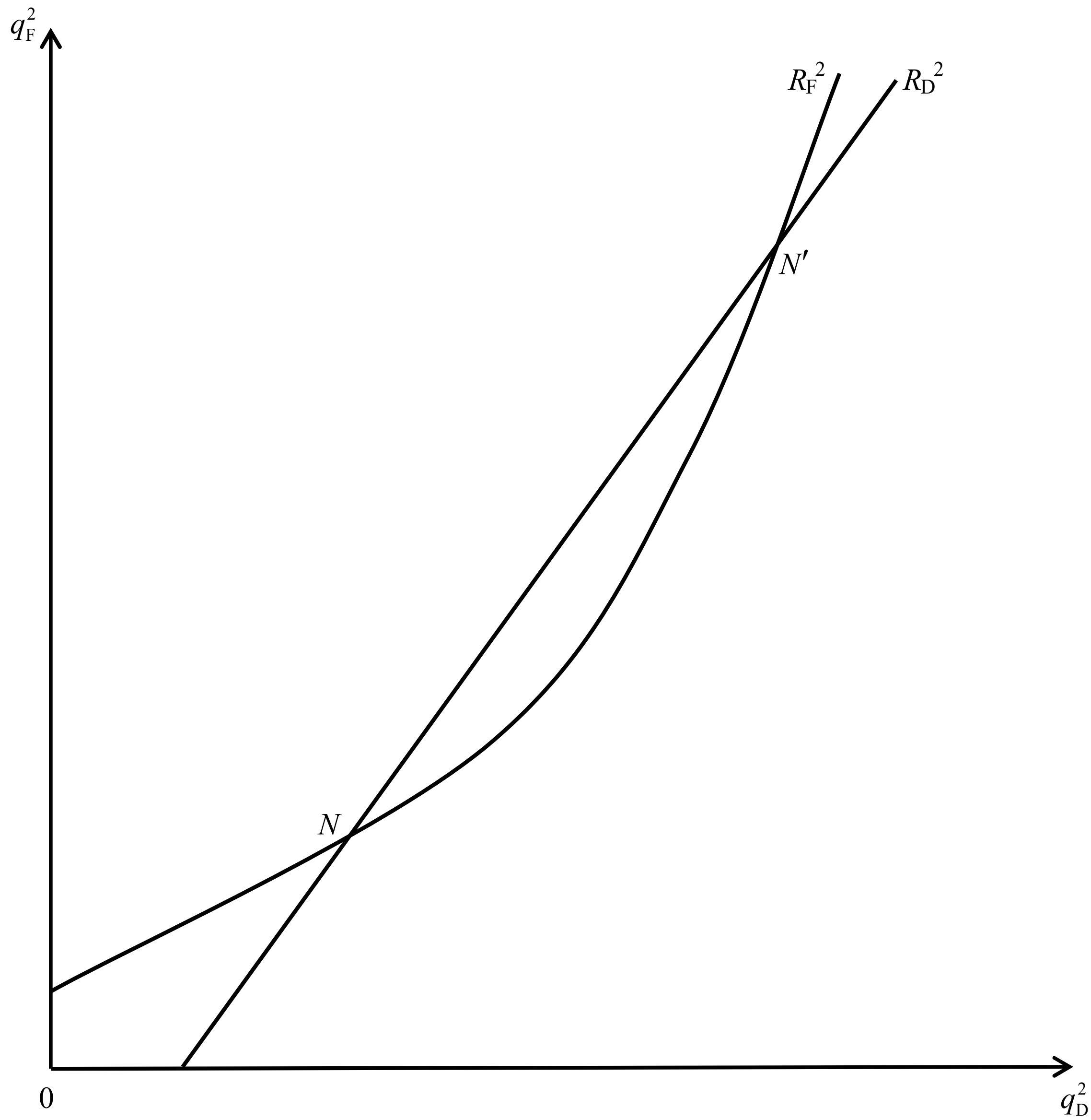

Figure 1: The reaction curves cross twice, but only point *N* is a stable Cournot equilibrium.

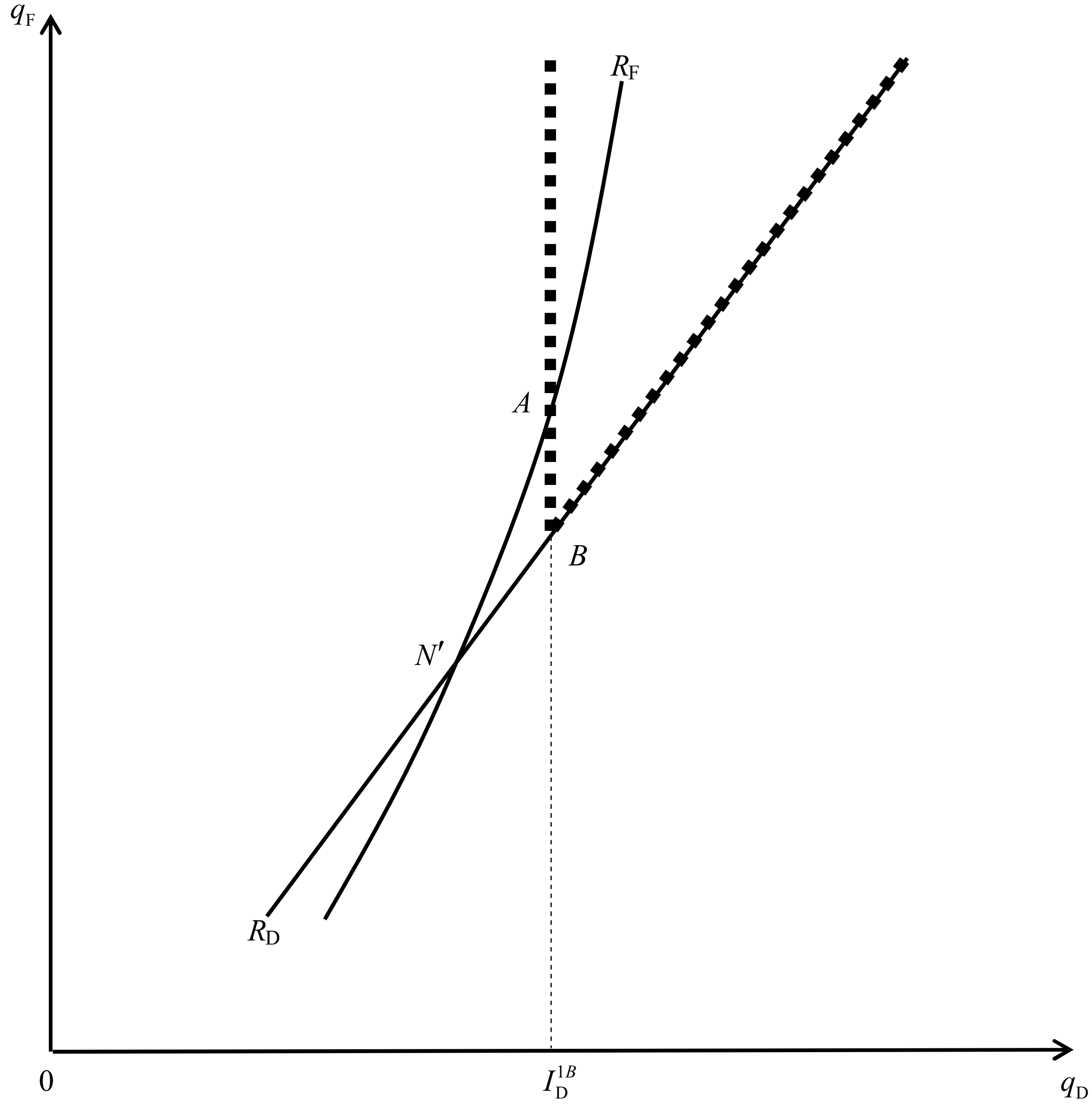

Figure 2: Firm D's best response is kinked at the level of I_D^{1B}.

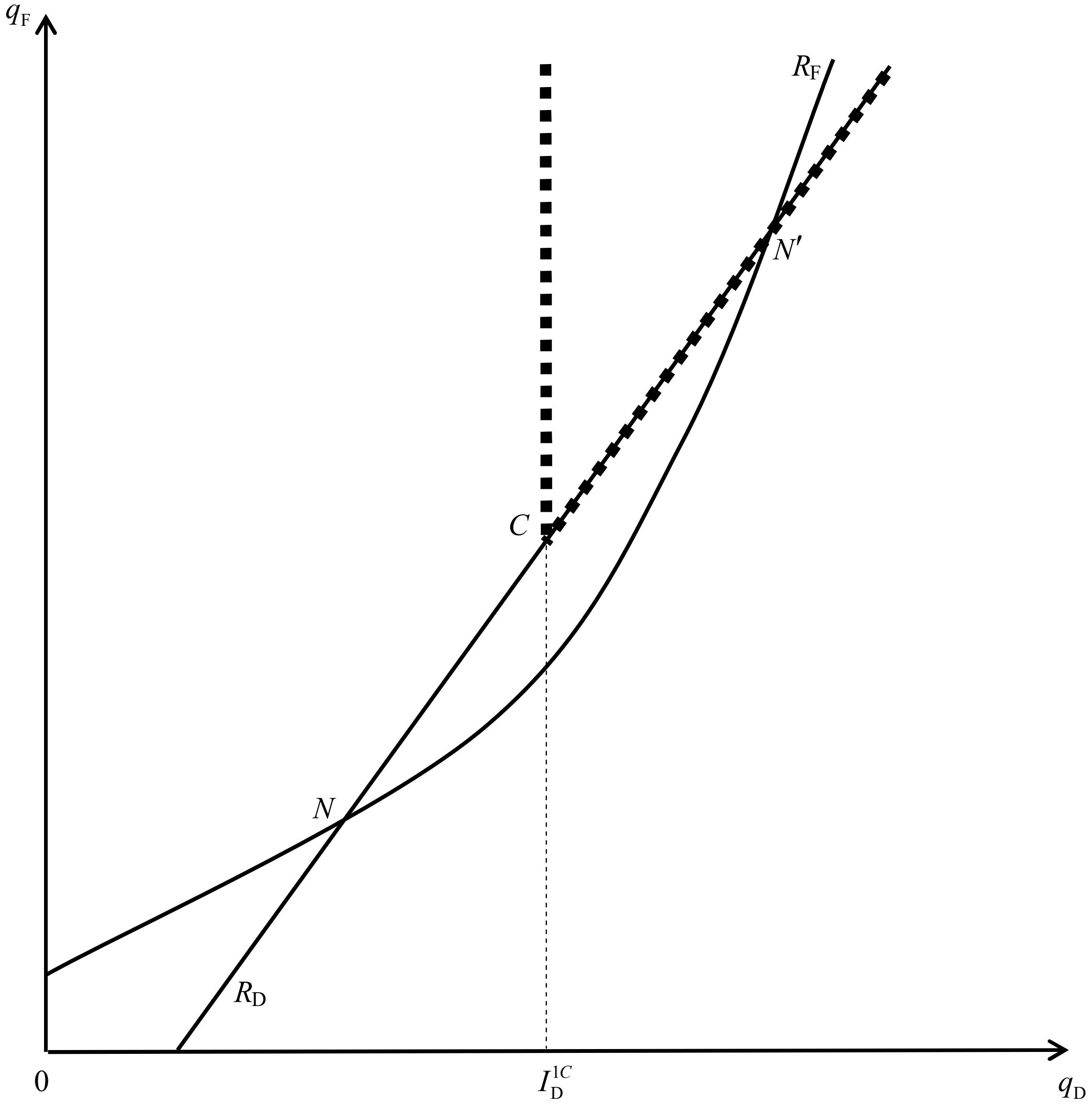

Figure 3: Point N' is not a stable solution.

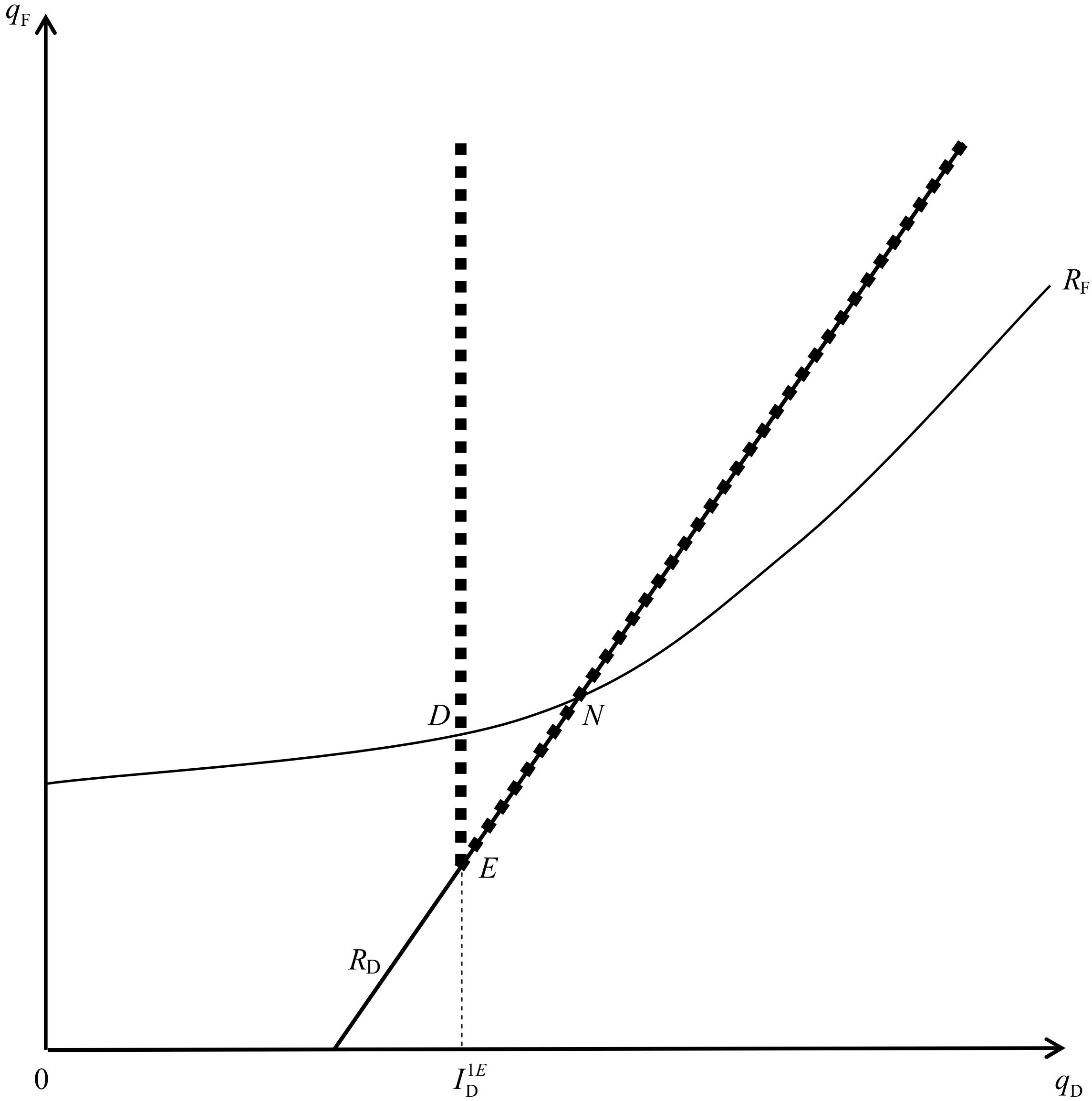

Figure 4: The intersection of new reaction curves is not affected by the kink.

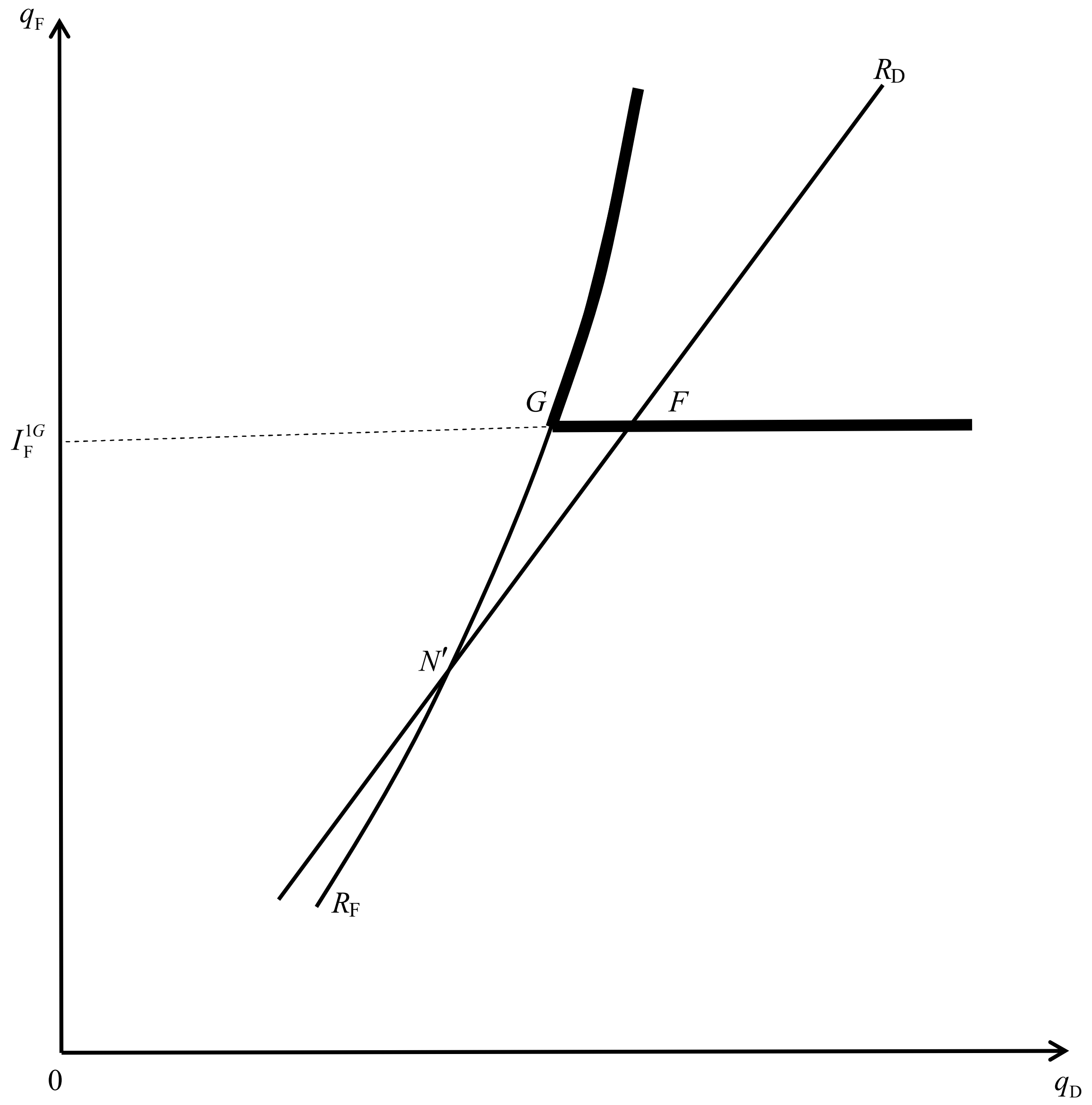

Figure 5: Firm F's best response is kinked at the level of I_F^{1G}.

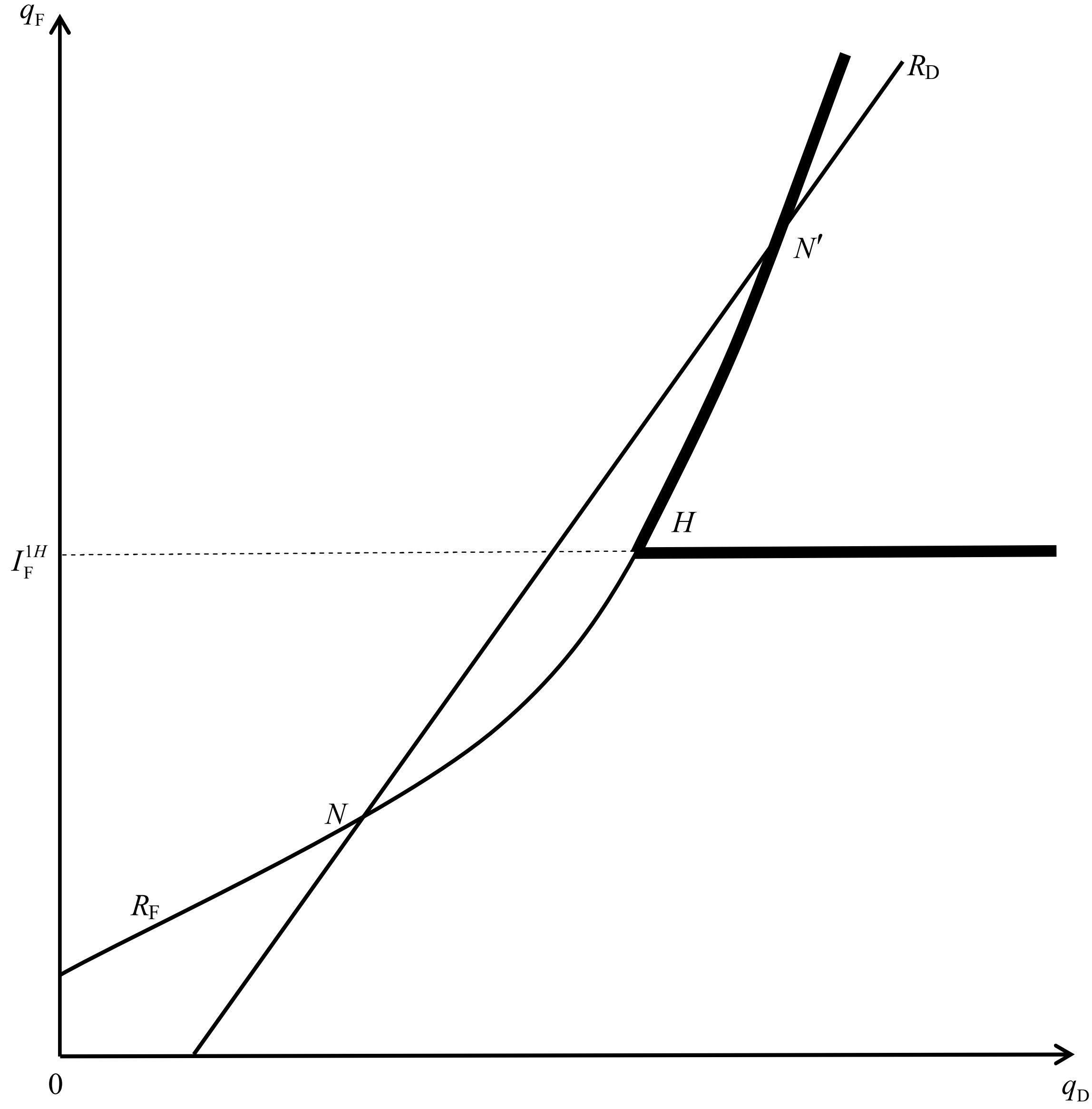

Figure 6: There is no stable solution.

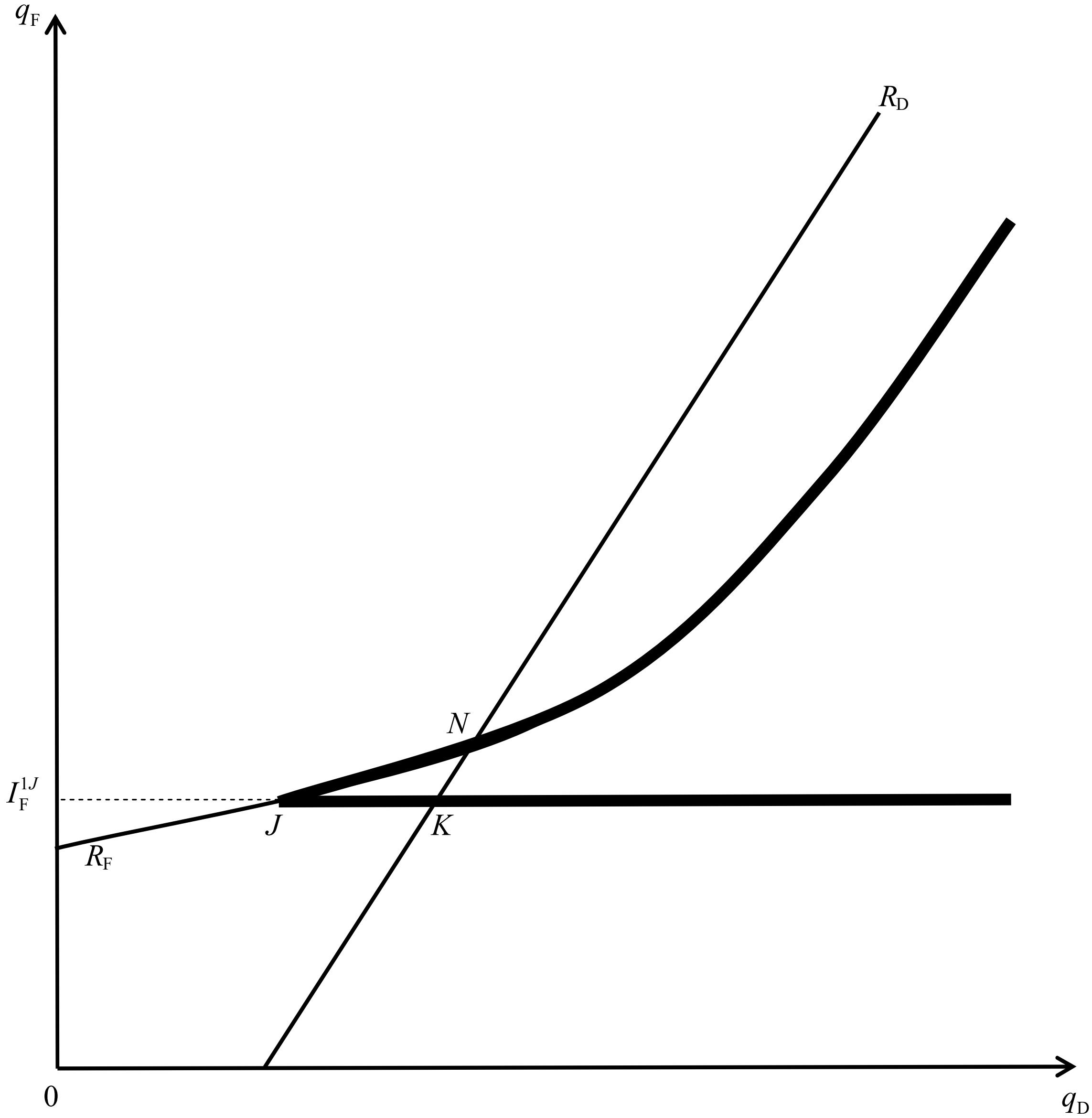

Figure 7: There are two stable solutions.

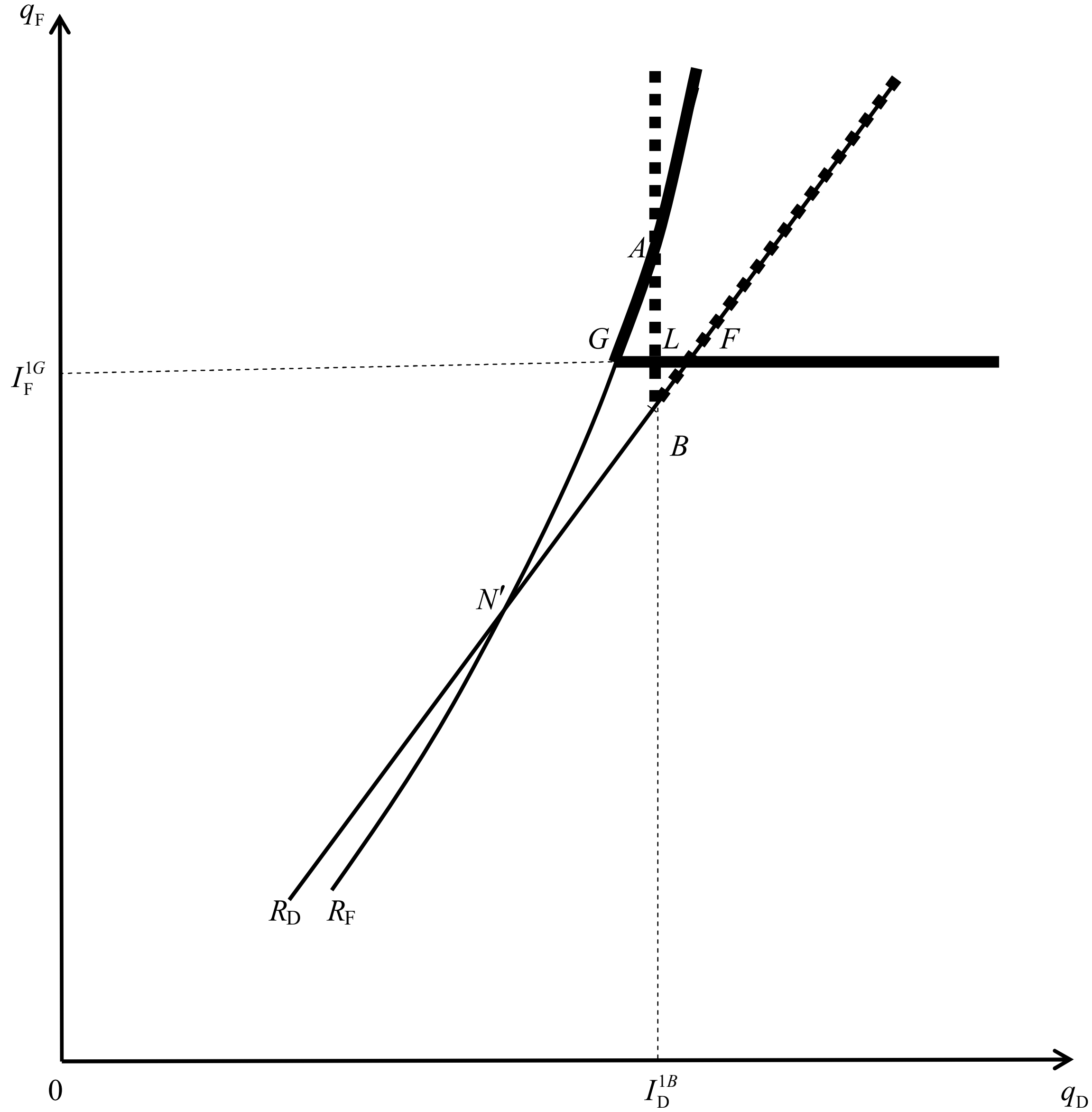

Figure 8: There are three stable solutions.

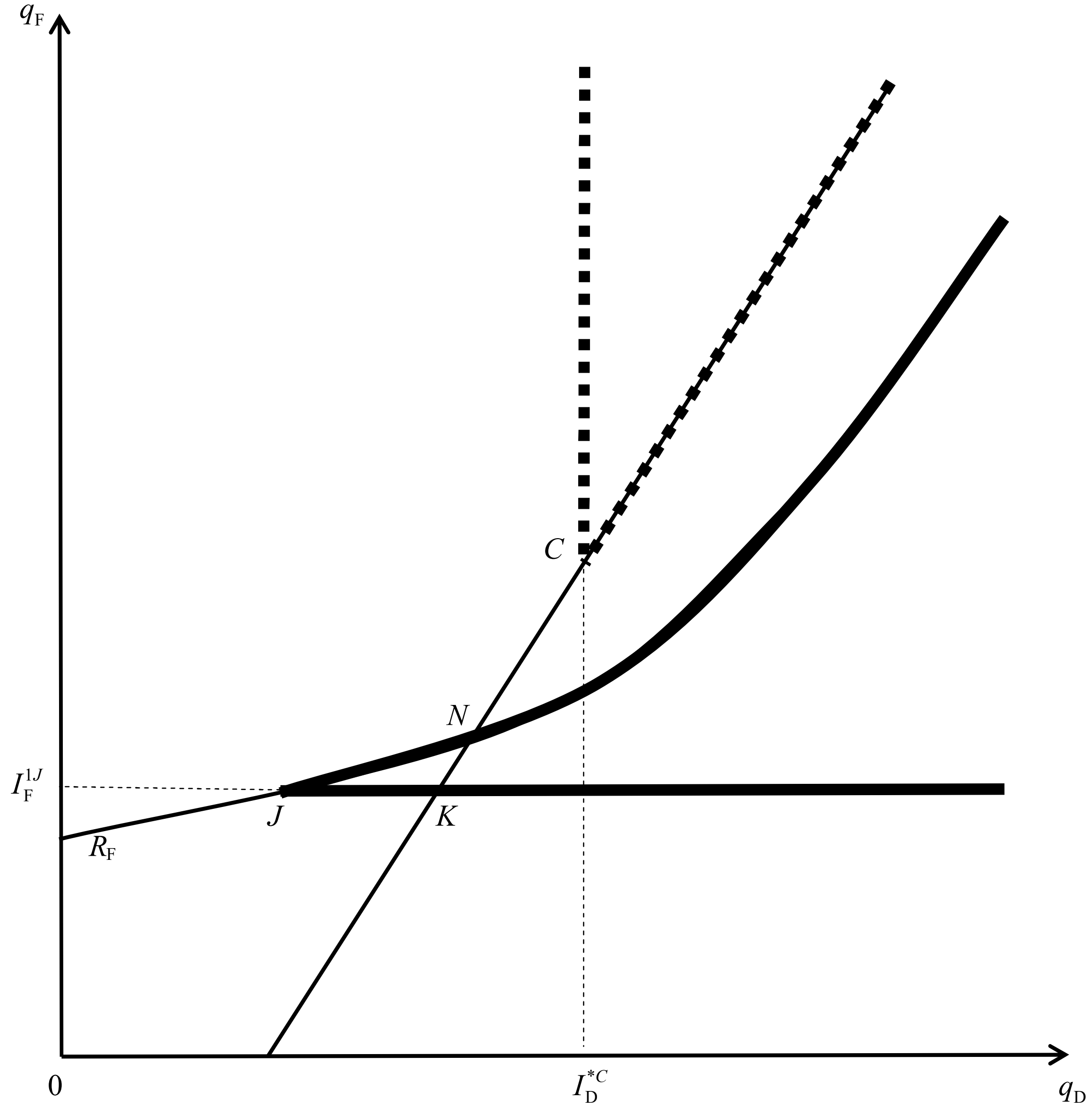

Figure 9: There is no stable solution.

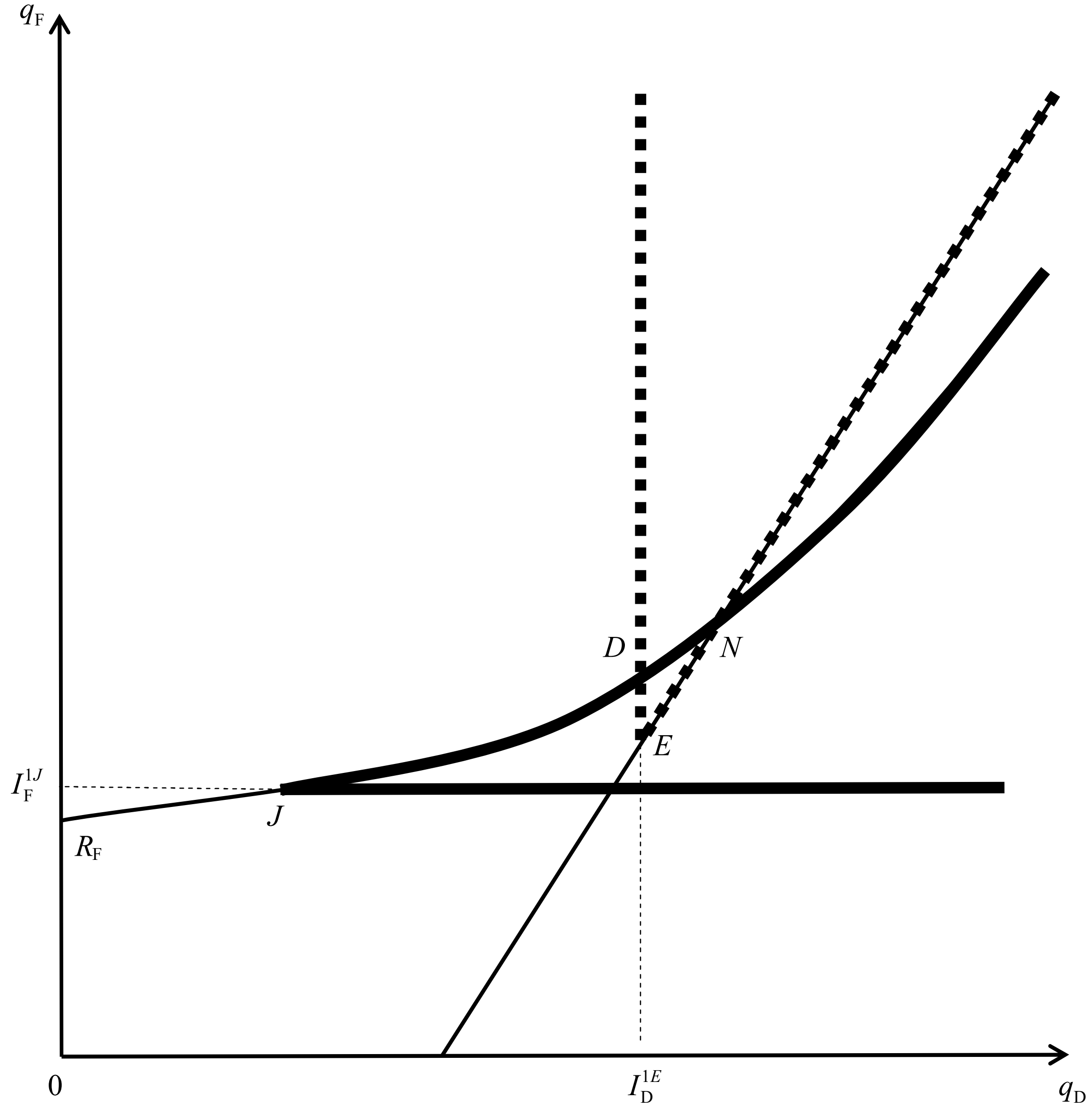

Figure 10: Both *N* and *D* are stable solutions.

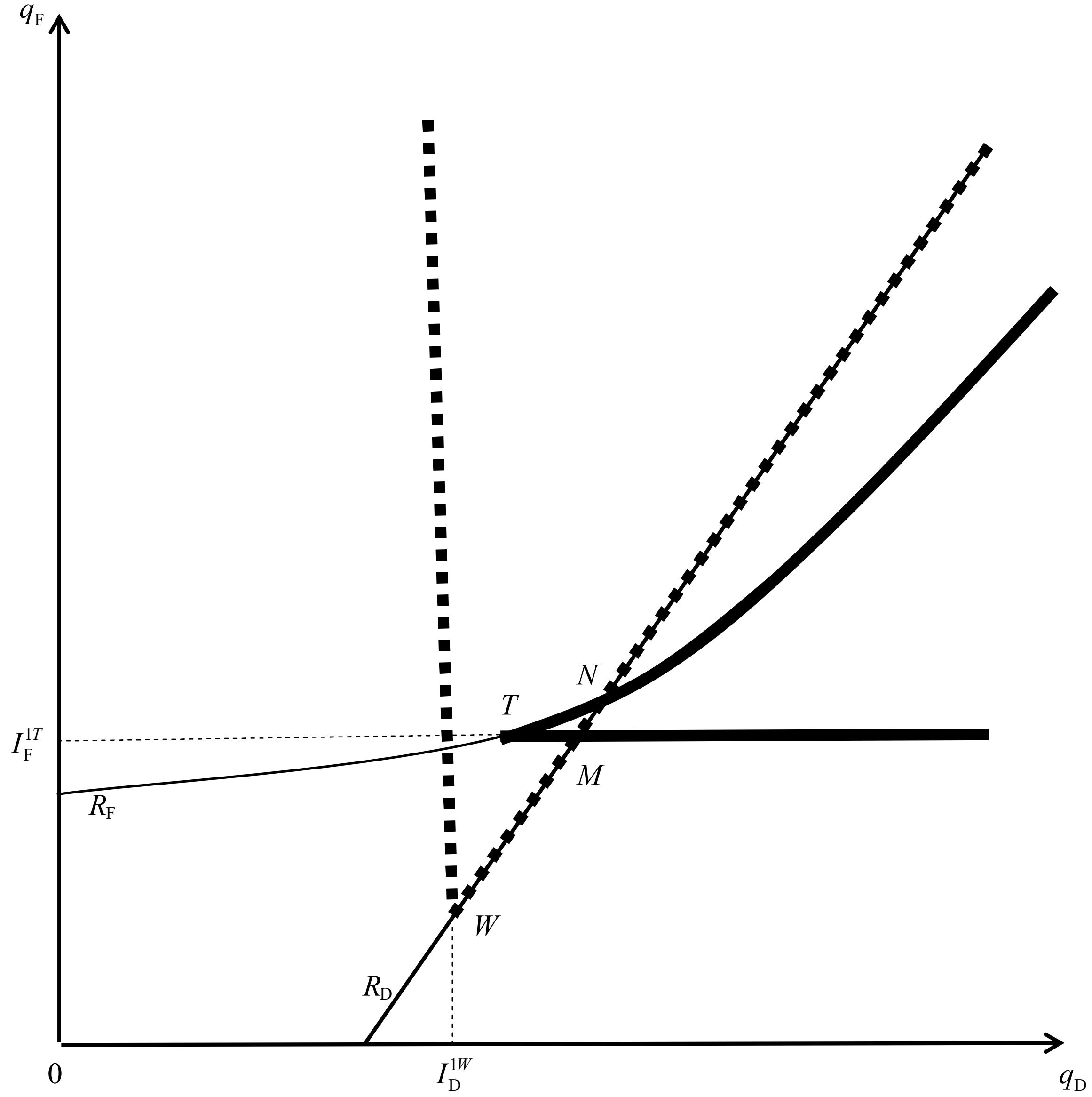

Figure 11: Both *N* and *M* are stable solutions.

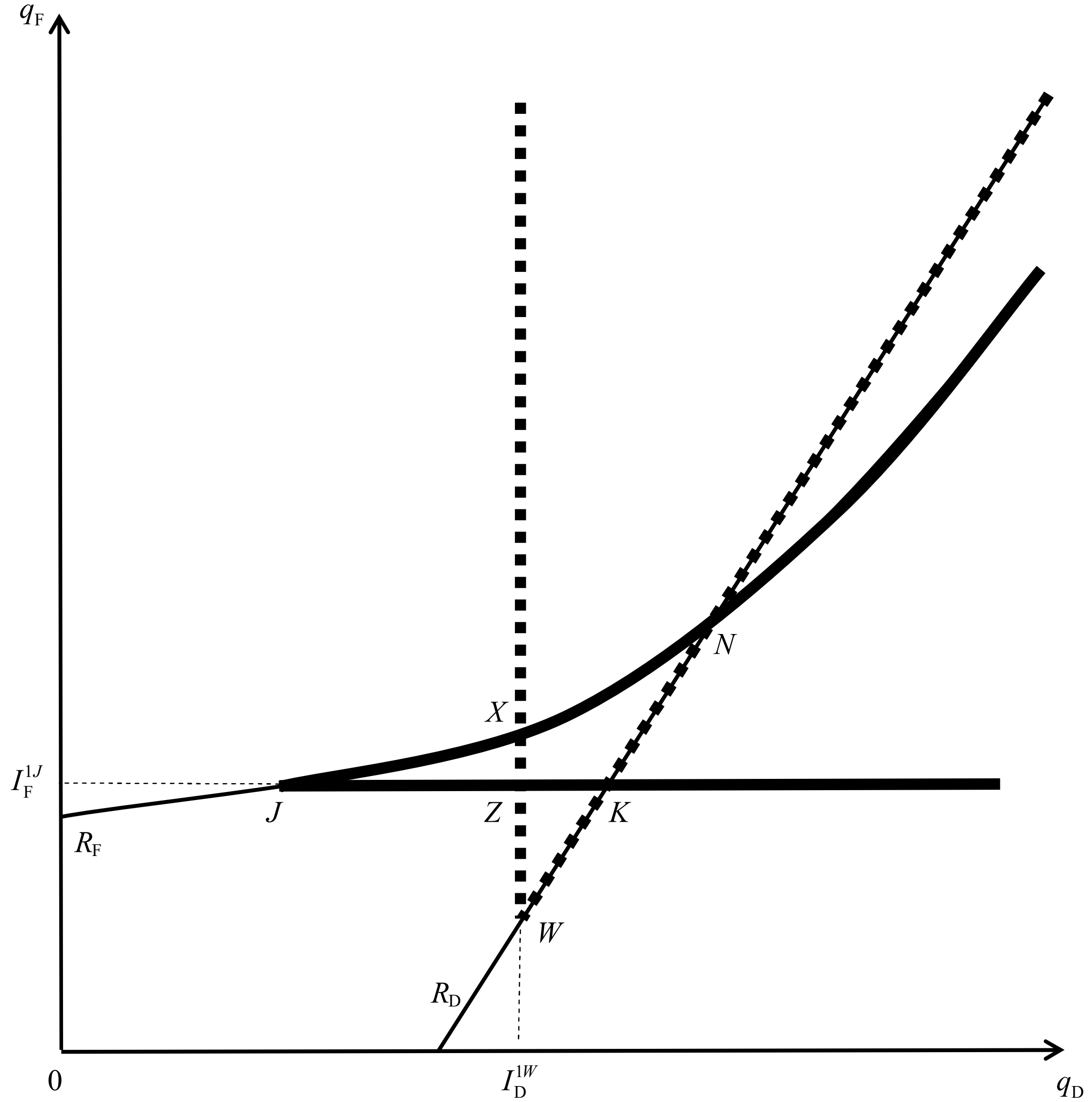

Figure 12: There are four stable solutions.