Population Growth and Trade Patterns in Semi-Endogenous Growth Economies

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Abstract

This paper builds a two-country, two-sector (manufacturing and agriculture), semi-endogenous growth model and investigates the relationship between trade patterns and the growth rate of per capita real consumption. Under free trade, if the home country produces both goods and the foreign country specializes in agriculture, then the per capita growth rates of the home country and foreign country are equalized. By contrast, if the home country specializes in manufacturing and the foreign country specializes in agriculture, then the per capita growth rate of the home country is higher than that of the foreign country.

Keywords: semi-endogenous growth model; trade patterns; population growth; per capita consumption growth

JEL Classification: F10; F43; O11; O41

1 Introduction

This paper builds a two-country, two-sector, semi-endogenous growth model and investigates the relationship between trade patterns and economic growth. We investigate how the per capita growth rate of a country changes depending on the sector in which it specializes.

Other studies have analyzed the relationship between trade patterns and growth.[1] Kaneko (2000) builds a growth model with human capital accumulation and shows that the relationship between the terms of trade and growth depends on whether the country specializes in

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[1] Wong and Yip (1999) present a small-open-economy, two-sector model of endogenous growth with capital accumulation and learning-by-doing and analyze the relationship between economic growth, industrialization, and international trade.

the consumption goods or the investment goods sector. If the home country specializes in the investment goods sector, its growth rate does not depend on the terms of trade. On the contrary, if the home country specializes in the consumption goods sector, its growth rate does depend on the terms of trade and increases as the terms of trade improve. However, Kaneko (2000) utilizes a small-open-economy model and hence the terms of trade are given exogenously.

Kaneko (2003) builds a two-country, two-sector, AK growth model and endogenizes the terms of trade. The author finds that if a country with a growth rate lower than that of its trade partner under autarky has a comparative advantage in the consumption goods sector, then the country can narrow or even reverse the growth gap by trading with the other country.

Felbermayr (2007) describes the situation where a capital-abundant North and a capital-scarce South trade with each other. In the model, the trade pattern is determined endogenously, while the North produces investment goods and the South produces consumption goods. The production technology of investment goods is determined by an AK model and that of consumption goods is based on a decreasing returns to scale model. Along the balanced growth path (BGP), the South's terms of trade are continuously improving, such that even its decreasing returns to scale can grow at the same rate as the North. Therefore, the South can eliminate the growth gap by trading.

The above studies use scale-growth models in which population size positively affects per capita growth. This assumption, however, seems counterfactual. Jones (1995) attempts to remove the scale effects by presenting a semi-endogenous growth model in which the growth rate of output per capita reacts positively to the population growth rate and not the size of the population. In other words, the higher the population growth rate, the faster the country grows.[2]

In this paper, we build a two-country, two-sector, semi-endogenous growth model in which manufacturing has increasing returns to scale and agriculture has constant returns to scale. We then investigate the relationship between trade patterns, growth, and income gaps between the two countries under free trade in the long run.

We use the semi-endogenous growth model for two reasons. First, we can obtain sustainable per capita income growth even though population growth is strictly positive. Second, we do not need to impose knife-edge conditions on the parameters of the model. To our knowledge, this model differs from most other models in that we explicitly consider population growth. In addition, existing models belong to the AK class of models, and as such,

[2]For a systematic exposition of scale effects and semi-endogenous growth, see Jones (1999, 2005), Aghion and Howitt (2005), and Dinopoulos and Sener (2007). For more sophisticated semi-endogenous growth models, see also Kortum (1997), Dinopoulos and Thompson (1998), Peretto (1998), Segerstrom (1998), Young (1998), Howitt (1999), and Dinopoulos and Syropoulos (2007).

impose knife-edge conditions on the production functions.

In this respect, Sasaki (2011a) builds a semi-endogenous growth, North-South economic development model and shows that along the BGP, both countries grow at the same rate but their per capita incomes grow at different rates because of the differences in population growth. In Sasaki (2011a), the growth rate of per capita consumption in the North may either be increasing or decreasing in Northern population growth, but it is increasing in Southern population growth, and the growth rate of per capita consumption in the South is decreasing in Southern population growth but is increasing in Northern population growth.

However, in Sasaki (2011a), the production pattern is fixed and given exogenously. By contrast, in the present paper, the trade pattern is determined endogenously. This modification has two advantages. First, we can examine whether the assumed trade pattern in Sasaki (2011a)—the low-population-growth North produces only manufactured goods, whereas the high-population-growth South produces only agricultural goods—is sustainable over time. Second, we can compare an autarkic situation with a free trade situation. In particular, we can investigate whether the growth rate of the per capita income of a country increases or decreases when it switches from autarky to free trade.

Our model is based on the small-open-economy model of Christiaans (2008). He extends Wong and Yip's (1999) model to develop a small-open-economy, semi-endogenous growth model in which agriculture has constant returns to scale and manufacturing has increasing returns to scale and examines the dynamics as the economy moves toward a long-run equilibrium. We extend Christiaans' small-open-economy model to a large two-country model. In this respect, Sasaki (2011b) is closely related to the present paper. Based on Christiaans (2008), Sasaki (2011b) builds a two-country, semi-endogenous growth model and investigates the relationship between long-run trade patterns and long-run per capita growth rates. However, the author only considers the case where the population growth rates are equal.

According to our analysis, we find that the difference between the population growth rates of the two countries affects the trade patterns and relationships between the per capita growth of the home country (Home hereafter) and that of the foreign country (Foreign hereafter).

Under autarky, the growth rate of per capita real consumption is higher in the country where population growth is higher than that of the other country, along the BGP. Under free trade, if Home diversifies, that is, produces both goods, and Foreign asymptotically specializes completely in agriculture,[3] then the BGP growth rates of Home and Foreign are equalized, and this trade pattern is sustainable as long as the population growth of Home

[3]The word "asymptotically" means that the agricultural output converges to zero, but it never vanishes because we assume that Foreign's capital stock is strictly positive. See also Christiaans (2008).

is higher than that of Foreign. On the contrary, under free trade, if Home specializes completely in manufacturing and Foreign asymptotically specializes completely in agriculture, then the BGP growth rate of Home is higher than that of Foreign, and this trade pattern is sustainable as long as the population growth of Home is lower than that of Foreign.

Therefore, the relationship between population growth and per capita consumption growth differs under autarky and free trade. Moreover, the magnitude of the relationship between the per capita consumption growth of Home and that of Foreign can be reversed under free trade.

We mention the effect on economic growth of population aging, which leads to a decline in population growth. Naito and Zhao (2009) examine how population aging affects trade patterns by formulating a two-country, two-good, two-factor, two-period-lived overlapping generations model in which the two countries are identical except for their exogenous rates of population growth.[4] In their model, good 1 is a capital good that is either invested or consumed, while good 2 is a pure consumption good. Both goods are produced with constant returns to scale production functions. They identify the aging (younger) country as the one with the lower (higher) exogenous rate of population growth and find that the low population growth aging country exports capital-intensive goods.

In our model, as shown later, the younger country with high population growth diversifies and produces manufactured goods, while the aging country with low population growth specializes in agriculture. That is, the younger country exports capital-intensive goods and hence our result is contrary to that of Naito and Zhao (2009). This difference lies in the specification of the production function of the manufacturing (capital goods-producing) sector. While they use a constant returns to scale production function, we adopt an increasing returns to scale production function.

The rest of the paper is organized as follows. Section 2 presents the framework of the model and analyzes the equilibrium under autarky. Section 3 describes the free trade equilibrium corresponding to each trade pattern and investigates whether each trade pattern is sustainable over time. Section 4 compares the growth rates of per capita real consumption under autarky and free trade in both countries. Section 5 concludes the paper.

2 The model

We consider a world that consists of two countries: Home and Foreign. Both countries produce homogeneous manufactured and agricultural goods. The manufactured good is

[4]There are very few studies that examine the effect of population aging on economic growth through channels of trade patterns. Naito and Zhao (2009), Sayan (2005), and Yakita (2012) are a few examples.

used for both consumption and investment, whereas the agricultural good is used only for consumption.

2.1 Production

Firms produce manufactured goods X_i^M with labor input L_i^M and capital stock K_i, and produce agricultural goods X_i^A with only labor input L_i^A. Here, $i = 1$ and $i = 2$ denote Home and Foreign, respectively. Both countries have the same production functions, which are specified as follows:[5]

$$
\begin{aligned}
X_i^M &= A_i K_i^{\alpha} (L_i^M)^{1-\alpha}, \quad \text{where } A_i = K_i^{\beta} && (1)\\
&= K_i^{\alpha+\beta} (L_i^M)^{1-\alpha}, \quad 0 < \alpha < 1,\ 0 < \beta < 1,\ \alpha + \beta < 1, && (2)\\
X_i^A &= L_i^A. && (3)
\end{aligned}
$$

Here, A_i in equation (1) represents an externality associated with capital accumulation, which captures the learning-by-doing effect à la Arrow (1962). By substituting A_i into equation (1), we obtain equation (2), which shows that the production of manufactured goods has increasing returns to scale, with β corresponding to the extent of the increasing returns.[6] Equation (3) shows that the production of agricultural goods has constant returns to scale.

Suppose that labor supply is equal to the population and that the population is fully employed. Moreover, suppose that the population grows at a constant rate n_i and the initial population is equal to unity in each country: $L_i(t) = L_i^M(t) + L_i^A(t) = e^{n_i t}$, with $n_i > 0$.

Let p_i denote the price of manufactured goods relative to agricultural goods. Then, the profits of manufacturing and agricultural firms are given by $\pi_i^M = p_i X_i^M - w_i L_i^M - p_i r_i K_i$ and $\pi_i^A = X_i^A - w_i L_i^A$, respectively, where w_i denotes the wage in terms of agricultural goods and

[5] The parameters of the production function show factor shares at competitive equilibrium. In the long run, factor shares do not differ so much for countries. Hence, we can justify using the same parameters for both countries. The same reasoning applies to the parameters of the utility functions that will be introduced below. Even if both countries' production functions have different parameters, we can investigate such situation. However, the analysis will become much complicated, and hence, it is very hard to obtain analytical solutions.

[6] We explain the derivation of the production function given by equation (1) as follows. Each of a large number of completely identical firms j of sector M uses labor L_{ij}^M and capital K_{ij} to produce its output X_{ij}^M according to the production function

$$X_{ij}^M = A_i K_{ij}^{\alpha} (L_{ij}^M)^{1-\alpha}, \quad A_i = K_i^{\beta}.$$

$L_i^M = \sum_j L_{ij}^M$, $K_i = \sum_j K_{ij}$, and $X_i^M = \sum_j X_{ij}^M$. Under perfect competition, the individual production functions j can be aggregated to a sectoral production function for the manufactured goods:

$$X_i^M = A_i K_i^{\alpha} (L_i^M)^{1-\alpha}.$$

r_i denotes the rental rate of capital.

From the profit-maximizing conditions, we obtain the following relations:

$$p_i \frac{\partial X_i^M}{\partial L_i^M} = w_i = 1, \tag{4}$$

$$\frac{\partial X_i^M}{\partial K_i} = r_i \text{ with } A_i \text{ given.} \tag{5}$$

From equation (4), we find that the wage is equal to unity as long as agricultural production is positive. We assume a Marshallian externality to derive equation (5). To analyze the market equilibrium, we use a Marshallian externality to make this externality compatible with perfect competition. In this case, an individual firm does not consider its own effect on the whole economy, and accordingly, it maximizes its profit without considering the effect of K_i on A_i; profit-maximizing firms regard A_i as given exogenously. Accordingly, firms do not internalize the effects of A_i.

2.2 Consumption

For simplicity, we make the classical assumption that wage and capital incomes are entirely devoted to consumption and saving, respectively.[7] In the canonical one-sector Solow model, under the golden rule of a steady state where per capita consumption is maximized, consumption is equal to real wages and all capital incomes are either saved or invested. Hence, our assumption has some rationality and it can be interpreted as a simple rule of thumb for consumers with dynamic optimization (Christiaans, 2008). We define real consumption per capita c_i as $c_i = C_i/L_i = (C_i^M)^\gamma (C_i^A)^{1-\gamma}/L_i$, where C_i denotes economy-wide real consumption. In this case, a proportion γ of wage income is spent on C_i^M and the rest, $1-\gamma$, is spent on C_i^A. Therefore, we obtain $p_i C_i^M = \gamma w_i L_i$ and $C_i^A = (1-\gamma) w_i L_i$. As explained later, the size of the expenditure coefficient γ affects the sustainability of the long-run trade patterns. We assume that both countries have an identical expenditure coefficient to simplify the analysis. Even when both countries have different expenditure coefficients, we can investigate this situation though the analysis will be complicated.

Moreover, the following relationship between real investment I_i and saving holds: $p_i I_i =$

[7] The same assumption is also used in Uzawa (1961), which considers a two-sector growth model, and Krugman (1981), which considers a two-country, two-sector, North-South trade and development model. Although consumption smoothing with dynamic optimization is a standard tool in macroeconomics, Mankiw (2000) states that consumption behavior deviates from the consumption-smoothed estimates.

$p_i r_i K_i$. From this equation, we obtain the rate of capital accumulation:

$$\frac{\dot{K}_i}{K_i} = r_i. \tag{6}$$

In other words, the rate of capital accumulation is equal to the rental rate of capital. A dot over a variable denotes the time derivative of the variable (e.g., $\dot{K}_i \equiv dK_i/dt$).

2.3 Equilibrium under autarky

Under autarky, both goods have to be produced.[8] The market-clearing conditions are as follows: $X_i^M = C_i^M + I_i$ and $X_i^A = C_i^A$. Note that $w_i = 1$ under autarky. From the market-clearing condition for manufactured goods, we obtain p_i, which is used to derive each sector's share of employment: $L_i^M/L_i = \gamma$ and $L_i^A/L_i = 1-\gamma$. Therefore, under autarky, each sector's share of employment is constant.

Under autarky, the relative price of manufactured goods is given by

$$p_i = \frac{(\gamma L_i)^\alpha}{(1-\alpha)K_i^{\alpha+\beta}}. \tag{7}$$

We now derive the BGP under autarky. Along the BGP, the rate of capital accumulation is constant and equal to the rental rate of capital, which is given from equation (5) by $r_i = \alpha K_i^{\alpha+\beta-1}(\gamma L_i)^{1-\alpha}$. From this, the BGP growth rates of K_i and p_i are given by

$$g_{K_i}^* = \frac{1-\alpha}{1-\alpha-\beta} n_i > 0, \tag{8}$$

$$g_{p_i}^* = -\frac{\beta}{1-\alpha-\beta} n_i < 0, \tag{9}$$

respectively, where $g_x \equiv \dot{x}/x$ denotes the growth rate of a variable x and an asterisk "$*$" denotes the value of the BGP.[9] The rate of capital accumulation is positive and proportionate to population growth, and the relative price of manufactured goods is decreasing at a constant rate.

Consumption is defined as consisting of wages only and hence the growth rate of per capita real consumption is equal to the growth rate of the real wage, given by $g_{c_i} = g_{w_i} - \gamma g_{p_i}$. Here, the real wage is obtained by deflating w_i by the consumer price index p_i^γ.[10] To obtain

[8] The derivation of the key equations is presented in the Appendix, which is available on request.

[9] The BGP under autarky is stable. The proof is given in the Appendix.

[10] Let p_c denote the consumer price index that is consistent with the expenditure minimization problem of consumers. Then, $p_c = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}p_i^\gamma$, and by definition, $p_c c_i = w_i$. Strictly speaking, the consumer price index is given by $\gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}p_i^\gamma$. However, we use p_i^γ because the constant terms have no effect on the

g_{c_i}, we must know g_{w_i} and g_{p_i}. Note that as long as agricultural goods are produced, the nominal wage is equal to unity, that is, $w_i = 1$, which means that $g_{w_i} = 0$. Accordingly, we obtain the growth rate of per capita consumption under autarky:

$$g_{c_i}^{AT} = \frac{\gamma\beta}{1-\alpha-\beta} n_i > 0, \tag{10}$$

where "AT" denotes autarky. Therefore, $g_{c_i}^{AT}$ is increasing in n_i. In addition, $g_{c_1}^{AT} \gtreqless g_{c_2}^{AT}$ according to $n_1 \gtreqless n_2$.

3 Equilibrium under free trade

Suppose that Home and Foreign engage in free trade at time zero.[11] If $K_1(0) > K_2(0)$, then from equation (7), $p_1(0) < p_2(0)$ because $L_1(0) = L_2(0) = 1$. Thus, if $K_1(0) > K_2(0)$, Home has a comparative advantage in manufactured goods and Foreign has a comparative advantage in agricultural goods at time zero. In the following analysis, we assume that $K_1(0) > K_2(0)$.

It is sufficient for us to consider the following four trade patterns from the viewpoint of Home:

Pattern 1 : Both countries produce both goods, that is, both countries diversify.
Pattern 2 : Home diversifies and Foreign specializes completely in agriculture.
Pattern 3 : Home specializes completely in manufacturing and Foreign specializes completely in agriculture.
Pattern 4 : Home specializes completely in manufacturing and Foreign diversifies.

3.1 Equilibrium when both countries diversify: Pattern 1

The market-clearing conditions for both goods are given by $X_1^M + X_2^M = C_1^M + C_2^M + I_1 + I_2$ and $X_1^A + X_2^A = C_1^A + C_2^A$. From these, we obtain

$$p^{\frac{1}{\alpha}} = \frac{\gamma(L_1 + L_2)}{(1-\alpha)^{\frac{1}{\alpha}}\left(K_1^{\frac{\alpha+\beta}{\alpha}} + K_2^{\frac{\alpha+\beta}{\alpha}}\right)}. \tag{11}$$

results.

[11] The derivation of the key equations is presented in the Appendix.

Each country's share of employment in the manufacturing sector, θ_i^M, is given by

$$\theta_1^M \equiv \frac{L_1^M}{L_1} = \frac{\gamma\left(1 + \frac{L_2}{L_1}\right)}{1 + \left(\frac{K_2}{K_1}\right)^{\frac{\alpha+\beta}{\alpha}}}, \quad \theta_2^M \equiv \frac{L_2^M}{L_2} = \frac{\gamma\left(1 + \frac{L_1}{L_2}\right)}{1 + \left(\frac{K_1}{K_2}\right)^{\frac{\alpha+\beta}{\alpha}}}. \tag{12}$$

The rates of capital accumulation in both countries are given by

$$g_{K_1} = \alpha K_1^{\alpha+\beta-1}(\theta_1^M L_1)^{1-\alpha}, \quad g_{K_2} = \alpha K_2^{\alpha+\beta-1}(\theta_2^M L_2)^{1-\alpha}. \tag{13}$$

First, if $n_1 = n_2$, so that $L_1 = L_2$,[12] then after enough time has passed, we obtain

$$\lim_{t\to+\infty} \theta_1^M = 2\gamma, \quad \lim_{t\to+\infty} \theta_2^M = 0, \tag{14}$$

where $\gamma < 1/2$ should be imposed. The share of employment in manufacturing in Foreign goes to zero, and Foreign asymptotically specializes completely in agriculture. Hence, Pattern 1 is unsustainable when $n_1 = n_2$.

Second, if $n_1 > n_2$, after enough time has passed, we obtain

$$\lim_{t\to+\infty} \theta_1^M = \gamma, \quad \lim_{t\to+\infty} \theta_2^M = 0. \tag{15}$$

In this case too, the share of employment in manufacturing in Foreign goes to zero and Foreign asymptotically specializes completely in agriculture. Hence, Pattern 1 is also unsustainable when $n_1 > n_2$.

Third, if $n_1 < n_2$, after enough time has passed, we obtain

$$\lim_{t\to+\infty} \theta_1^M = +\infty, \quad \lim_{t\to+\infty} \theta_2^M = 0. \tag{16}$$

In this case, the share of employment in manufacturing in Home exceeds unity, the share of employment in manufacturing in Foreign goes to zero, and Foreign asymptotically specializes completely in agriculture. Hence, Pattern 1 is also unsustainable when $n_1 < n_2$.

Summarizing the above results, we obtain the following proposition.

Proposition 1. *Pattern 1 is unsustainable irrespective of the sizes of* n_1 *and* n_2.

As stated in the Introduction, Sasaki (2011b) considers this trade pattern when $n_1 = n_2$. Even in that case, the long-run equilibrium is a saddle point and thereby unstable. Accordingly, Pattern 1 is completely unstable irrespective of the sizes of n_1 and n_2.

[12]In addition to $n_1 = n_2$, if $K_1(0) = K_2(0)$, the share of employment in manufacturing in each country is given by $\theta_i^M = \gamma$, which is constant. Pattern 1 is sustainable only in this case. However, the relative prices in both countries under autarky are equal and therefore trade does not occur.

3.2 Equilibrium when Home diversifies and Foreign specializes in agriculture: Pattern 2

The market-clearing conditions for both goods are given by $X_1^M = C_1^M + C_2^M + I_1$ and $X_1^A + X_2^A = C_1^A + C_2^A$. From these, we obtain

$$p^{\frac{1}{\alpha}} = \frac{\gamma(L_1 + L_2)}{(1-\alpha)^{\frac{1}{\alpha}} K_1^{\frac{\alpha+\beta}{\alpha}}}. \tag{17}$$

The share of employment in manufacturing in Home is given by

$$\theta_1^M = \gamma\left(1 + \frac{L_2}{L_1}\right). \tag{18}$$

First, if $n_1 = n_2 = n$, the share of employment in manufacturing in Home becomes

$$\theta_1^M = 2\gamma. \tag{19}$$

In this case, we need $\gamma < 1/2$ for Pattern 2 to hold.

Second, if $n_1 > n_2$, we obtain

$$\lim_{t\to+\infty} \theta_1^M = \gamma. \tag{20}$$

The share of employment in manufacturing in Home converges to γ.

Third, if $n_1 < n_2$, then θ_1^M continues to increase, becoming more than unity and approaches infinity.

$$\lim_{t\to+\infty} \theta_1^M = +\infty. \tag{21}$$

Thus, Pattern 2 is unsustainable.

The growth rate of capital stock is given by

$$g_{K_1} = \alpha\gamma^{1-\alpha}(L_1 + L_2)^{1-\alpha} K_1^{\alpha+\beta-1}. \tag{22}$$

Note that in this case, we obtain $c_1 = c_2$ because $w_1 = w_2 = 1$ as long as agricultural goods are produced and both countries face the same relative price p. Accordingly, in Pattern 2, the long-run growth rates of per capita consumption in Home and Foreign are equalized, that is, $g_{c_1}^{FT} = g_{c_2}^{FT}$, where "FT" denotes free trade.

We now examine in detail the conditions under which Pattern 2 holds. Following Wong and Yip (1999), we investigate whether the trade pattern is sustainable by comparing the size of the terms of trade with the marginal rate of transformation (MRT) of the production possibilities frontier (PPF) at the corner point where a country specializes completely in manufacturing (see Figure 1).

[Figure 1 around here]

The size of the MRT of the PPF in Home is given by

$$-\frac{dX_1^A}{dX_1^M} = \frac{[K_1^{\alpha+\beta}(\theta_1^M L_1)^{1-\alpha}]^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)K_1^{\frac{\alpha+\beta}{1-\alpha}}}. \tag{23}$$

By substituting $\theta_1^M = 1$ into equation (23), the size of the MRT at the point where Home specializes completely in manufacturing is given by

$$\bar{\chi}_1 = \frac{L_1^\alpha}{(1-\alpha)K_1^{\alpha+\beta}}. \tag{24}$$

For Pattern 2 to be sustainable over time, we need $p < \bar{\chi}_1$, that is,

$$\frac{[\gamma(L_1 + L_2)]^\alpha}{(1-\alpha)K_1^{\alpha+\beta}} < \frac{L_1^\alpha}{(1-\alpha)K_1^{\alpha+\beta}}. \tag{25}$$

By rearranging this condition, we obtain

$$L_2 < \frac{1-\gamma}{\gamma} L_1. \tag{26}$$

From this, if $n_1 = n_2$, Pattern 2 is sustainable if $\gamma < 1/2$. On the contrary, if $n_1 > n_2$, Pattern 2 is sustainable in the long run irrespective of the size of γ. By contrast, if $n_1 < n_2$, Pattern 2 is unsustainable in the long run.

However, we must also consider another condition to ascertain the sustainability of Pattern 2, that is, the relationship between the MRT of the PPF of Foreign and the terms of trade. The size of the MRT at the point where Foreign specializes completely in manufacturing is given by

$$\bar{\chi}_2 = \frac{L_2^\alpha}{(1-\alpha)K_2^{\alpha+\beta}}. \tag{27}$$

For Pattern 2 to be sustainable in the long run, it is necessary for $p < \bar{\chi}_2$.

If $n_1 \geq n_2$, we find that $p < \bar{\chi}_2$ holds over time because $g_p = -\frac{\beta}{1-\alpha-\beta} n_1 < 0$ and $g_{\bar{\chi}_2} = \alpha n_1 > 0$ in the long run. Therefore, Pattern 2 is sustainable.

Summarizing the above results, we obtain the following proposition.

Proposition 2. *If $n_1 = n_2$, Pattern 2 is sustainable as long as $\gamma < 1/2$. If $n_1 > n_2$, Pattern 2 is sustainable over time. By contrast, if $n_1 < n_2$, Pattern 2 is unsustainable in the long run.*

This pattern is unsustainable in the long run when $n_1 < n_2$ because demand for manufactured goods in Foreign grows faster than the supply of Home; hence, Home alone cannot meet global demand for manufactured goods.

Now, we derive the BGP growth rates. By differentiating equation (22) with respect to time, we obtain the growth rate of g_{K_1}:

$$\frac{\dot{g}_{K_1}}{g_{K_1}} = (1-\alpha)\left(\frac{L_1}{L_1+L_2}n_1 + \frac{L_2}{L_1+L_2}n_2\right) + (\alpha+\beta-1)g_{K_1}. \tag{28}$$

When $n_1 \geq n_2$, this leads to

$$\frac{\dot{g}_{K_1}}{g_{K_1}} = (1-\alpha)n_1 + (\alpha+\beta-1)g_{K_1}. \tag{29}$$

With $\dot{g}_{K_1}/g_{K_1} = 0$, the growth rates of capital stock, terms of trade, and per capita consumption are given by

$$g_{K_1} = \frac{1-\alpha}{1-\alpha-\beta}n_1 > 0, \tag{30}$$

$$g_p = -\frac{\beta}{1-\alpha-\beta}n_1 < 0, \tag{31}$$

$$g_{c_1}^{FT} = g_{c_2}^{FT} = \frac{\gamma\beta}{1-\alpha-\beta}n_1. \tag{32}$$

Accordingly, both countries grow at a rate that is increasing in n_1.

3.3 Equilibrium when Home specializes in manufacturing and Foreign specializes in agriculture: Pattern 3

The market-clearing conditions for both goods are given by $X_1^M = C_1^M + C_2^M + I_1$ and $X_2^A = C_1^A + C_2^A$. With $L_1^M = L_1$, we obtain

$$p = \frac{\gamma L_2}{(1-\alpha)(1-\gamma)K_1^{\alpha+\beta}L_1^{1-\alpha}}. \tag{33}$$

The growth rate of capital stock is given by

$$g_{K_1} = \alpha K_1^{\alpha+\beta-1} L_1^{1-\alpha}. \tag{34}$$

Pattern 3 is sustainable if $p > \bar{\chi}_1$, which can be rewritten as

$$\frac{\gamma L_2}{(1-\alpha)(1-\gamma)K_1^{\alpha+\beta} L_1^{1-\alpha}} > \frac{L_1^{\alpha}}{(1-\alpha)K_1^{\alpha+\beta}}. \tag{35}$$

From this, we obtain

$$L_2 > \frac{1-\gamma}{\gamma} L_1. \tag{36}$$

From this, if $n_1 = n_2$, Pattern 3 is sustainable as long as $\gamma > 1/2$. If $n_1 < n_2$, Pattern 3 is sustainable in the long run irrespective of the size of γ. On the contrary, if $n_1 > n_2$, Pattern 3 is unsustainable in the long run. In summary, $n_1 \leq n_2$ is a necessary condition for Pattern 3 to be sustainable.

However, as in Pattern 2, we must also consider another condition. For Pattern 3 to be sustainable in the long run, we need $p < \bar{\chi}_2$.

If $n_1 = n_2$, we obtain $g_p = -\frac{\beta}{1-\alpha-\beta} n_1 < 0$ and $g_{\bar{\chi}_2} = \alpha n_1 > 0$, which shows that $p < \bar{\chi}_2$ holds over time; hence, Pattern 3 is sustainable.

If $n_1 < n_2$, we have $g_p = n_2 - \frac{1-\alpha}{1-\alpha-\beta} n_1$ and $g_{\bar{\chi}_2} = \alpha n_2 > 0$. In this case, we obtain

$$|g_{\bar{\chi}_2}| - |g_p| = -(1-\alpha)\left(n_2 - \frac{1}{1-\alpha-\beta} n_1\right). \tag{37}$$

If $n_2 < \frac{1-\alpha}{1-\alpha-\beta} n_1$, that is, $g_p < 0$, Pattern 3 is sustainable. If $n_2 \geq \frac{1-\alpha}{1-\alpha-\beta} n_1$, that is, $g_p \geq 0$, we need $n_2 \leq \frac{1}{1-\alpha-\beta} n_1$ for $|g_{\bar{\chi}_2}| \geq |g_p|$ to hold. Therefore, if $\frac{1-\alpha}{1-\alpha-\beta} n_1 \leq n_2 \leq \frac{1}{1-\alpha-\beta} n_1$, Pattern 3 is sustainable. On the contrary, if $\frac{1}{1-\alpha-\beta} n_1 < n_2$, Pattern 3 is unsustainable.

Summarizing the above results, we obtain the following proposition.

Proposition 3. *If $n_1 = n_2$, Pattern 3 is sustainable over time as long as $\gamma > 1/2$. If $n_1 > n_2$, Pattern 3 is unsustainable in the long run. If $n_1 < n_2$, Pattern 3 is sustainable as long as $n_2 \leq \frac{1}{1-\alpha-\beta} n_1$*

This pattern is unsustainable in the long run when $n_1 > n_2$ because demand for agricultural goods in Home grows faster than supply in Foreign; hence, Foreign alone cannot meet global demand for agricultural goods.

Note that in this case, we obtain $c_1 > c_2$ because $w_1 = [\gamma/(1-\gamma)] \cdot (L_2/L_1) > 1 > w_2 = 1$ from equation (36) and both countries face the same relative price p.

Now, we derive the BGP growth rates.

$$g_{K_1} = \frac{1-\alpha}{1-\alpha-\beta} n_1 > 0, \tag{38}$$

$$g_p = n_2 - \frac{1-\alpha}{1-\alpha-\beta} n_1, \tag{39}$$

$$g_{c_1}^{FT} = (n_2 - n_1) - \gamma\left(n_2 - \frac{1-\alpha}{1-\alpha-\beta} n_1\right) = \frac{\beta-(1-\gamma)(1-\alpha)}{1-\alpha-\beta} n_1 + (1-\gamma)n_2, \tag{40}$$

$$g_{c_2}^{FT} = \frac{\gamma(1-\alpha)}{1-\alpha-\beta} n_1 - \gamma n_2. \tag{41}$$

Accordingly, $g_{c_1}^{FT}$ is increasing in n_1 if $\beta > (1-\gamma)(1-\alpha)$, decreasing in n_1 if $\beta < (1-\gamma)(1-\alpha)$, and increasing in n_2. In addition, $g_{c_2}^{FT}$ is increasing in n_1 and decreasing in n_2. In standard semi-endogenous growth models, the growth rate of per capita consumption (income) is increasing in the population growth rate. However, in our model, Home's per capita consumption growth can be increasing or decreasing in Home's population growth, and Foreign's per capita consumption growth is decreasing in Foreign's population growth. These results are consistent with the empirical evidence that indicates that in developed countries, the correlation between per capita income growth and population growth is ambiguous, whereas in developing countries, the correlation is negative (Sasaki, 2011a).

3.4 Equilibrium when Home specializes in manufacturing and Foreign diversifies: Pattern 4

The market-clearing conditions for both goods are given by $X_1^M + X_2^M = C_1^M + C_2^M + I_1 + I_2$ and $X_2^A = C_1^A + C_2^A$. From these equations, we find that the terms of trade satisfy the following equation:

$$(1-\alpha)^{\frac{1}{\alpha}} p^{\frac{1}{\alpha}} K_2^{\frac{\alpha+\beta}{\alpha}} = \gamma L_2 - (1-\alpha)(1-\gamma) p K_1^{\alpha+\beta} L_1^{1-\alpha}. \tag{42}$$

From this equation, p is implicitly and uniquely determined and hence p is a function of K_1, K_2, L_1, and L_2: $p = p(K_1, K_2, L_1, L_2)$.[13]

The growth rates of capital stock in each country are given by

$$g_{K_1} = \alpha K_1^{\alpha+\beta-1} L_1^{1-\alpha}, \quad g_{K_2} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} p^{\frac{1-\alpha}{\alpha}} K_2^{\frac{\beta}{\alpha}}, \tag{43}$$

[13] The left-hand side of equation (42) is an increasing function of p, whereas the right-hand side is a decreasing function of p. By plotting both functions, we find that the intersection of the functions is unique and gives an instantaneous equilibrium value of p.

where p is endogenously determined by equation (42).

The share of employment in manufacturing in Foreign is given by

$$\theta_2^M = \frac{(1-\alpha)^{\frac{1}{\alpha}} p^{\frac{1}{\alpha}} K_2^{\frac{\alpha+\beta}{\alpha}}}{L_2}. \tag{44}$$

In this case, analytical solutions are hard to obtain and thus we conduct numerical simulations.[14] From these simulations, we find that regardless of whether $n_1 \gtreqless n_2$, the share of employment in manufacturing in Foreign tends to zero in a finite time, that is, $\theta_2^M \to 0$, from which we obtain the following proposition.

Proposition 4. *Pattern 4 is unsustainable in the long run irrespective of the sizes of n_1 and n_2.*

The production of manufactured goods has increasing returns to scale. Thus, if both countries produce manufactured goods, global demand for manufactured goods will be unable to meet global supply of manufactured goods. Therefore, Pattern 4 is unsustainable in the long run.

3.5 Transitional dynamics

In this subsection, we briefly investigate the transitional dynamics. Suppose that each country is located on its BGP and that at some point in time, both countries switch from autarky to free trade (and they continue to engage in free trade).

Under the autarkic BGP, both countries produce both goods (i.e., they both diversify). From our assumption, Home has a comparative advantage in manufacturing, while Foreign has a comparative advantage in agriculture.

First, we consider the situation in which the population growth rate of Home is higher than that of Foreign ($n_1 > n_2$). When switching to free trade, both countries diversify (Pattern 1). However, from Proposition 1, Pattern 1 is unsustainable in the long run. In the case of $n_1 > n_2$, Foreign's comparative advantage in agriculture continues to intensify over time, and it experiences structural change in the sense that it continues to turn into an agricultural country. However, when $n_1 > n_2$, Foreign's supply of agricultural goods alone cannot meet global demand for agriculture and hence Home also has to produce agricultural goods. Therefore, in the long run, Home produces both goods, while Foreign completely specializes in agriculture (Pattern 2).

Second, we consider the situation in which the population growth rate of Home is lower than that of Foreign ($n_1 < n_2$). As in the first case, when switching to free trade, both

[14]The methods and results of the numerical simulations are presented in the Appendix.

countries produce both goods but this diversification is unsustainable in the long run. In the case of $n_1 < n_2$, Home's (Foreign's) comparative advantage in manufacturing (agriculture) continues to intensify over time, and Home (Foreign) continues to turn into a manufacturing (agricultural) country. In other words, both countries experience structural change. When $n_1 < n_2$, Foreign's supply of agricultural goods alone can meet global demand for agriculture and hence both countries' complete specialization is sustainable in the long run (Pattern 3).

3.6 Comparison with Christiaans' (2008) model

In this subsection, we compare our results of trade patterns with those obtained from Christiaans' (2008) small-open-economy model. Suppose that Home is assumed to be too small to affect the rest of the world (ROW), meaning that it receives the relative global price p_w as a given. Suppose also that the structure of the ROW is identical to that of Home, except for the population growth rate n_w, and that the ROW is on its BGP, where it produces both manufactured and agricultural goods. This implies that p_w is decreasing at a constant rate of $g^*_{p_w} = -\frac{\beta}{1-\alpha-\beta} n_w < 0$.

From this analysis, two sustainable trade patterns are obtained. First, if $n < n_w$, then Home asymptotically specializes completely in agriculture. Second, if $n > n_w$, then Home specializes completely in manufacturing.

In Pattern 2, Home diversifies and Foreign specializes in agriculture when $n_1 > n_2$. Accordingly, if we regard Home as the ROW and Foreign as Home, the result of our two-country model is consistent with that of Christiaans' small-open-economy model. In other words, a country with low population growth specializes in agriculture, while a country with high population growth diversifies.

In Pattern 3, Home specializes in manufacturing and Foreign specializes in agriculture when $n_1 < n_2$; hence, both countries specialize completely. By contrast, in Christiaans' model, when Home specializes in manufacturing, the ROW diversifies. Accordingly, our result that both countries specialize completely is a unique characteristic of our two-country model.

4 Per capita consumption growth rates under free trade

From the above analysis, we find that Patterns 2 and 3 are sustainable trade patterns. In this section, we summarize the effect of population growth on the long-run growth rate of per capita consumption under free trade.

4.1 Case 1: $n_1 = n_2$

If $0 < \gamma < 1/2$, only Pattern 2 is sustainable, and if $1/2 < \gamma < 1$, only Pattern 3 is sustainable. In both cases, the BGP growth rates of per capita consumption are given by

$$g_{c_1}^{FT} = g_{c_2}^{FT} = \frac{\gamma\beta}{1-\alpha-\beta} n_1 > 0. \tag{45}$$

4.2 Case 2: $n_1 > n_2$

Only Pattern 2 is sustainable, and the BGP growth rates of per capita consumption are given by

$$g_{c_1}^{FT} = g_{c_2}^{FT} = \frac{\gamma\beta}{1-\alpha-\beta} n_1 > 0. \tag{46}$$

4.3 Case 3: $n_1 < n_2$

Only Pattern 3 is sustainable. The BGP growth rates of per capita consumption are given by

$$g_{c_1}^{FT} = (n_2 - n_1) - \gamma\left(n_2 - \frac{1-\alpha}{1-\alpha-\beta} n_1\right), \tag{47}$$

$$g_{c_2}^{FT} = \frac{\gamma(1-\alpha)}{1-\alpha-\beta} n_1 - \gamma n_2. \tag{48}$$

If $\frac{1-\alpha}{1-\alpha-\beta} n_1 \leq n_2 \leq \frac{1}{1-\alpha-\beta} n_1$, we have $g_{c_1}^{FT} > 0$ and $g_{c_2}^{FT} \leq 0$.[15] On the contrary, if $n_2 < \frac{1-\alpha}{1-\alpha-\beta} n_1$, we have $g_{c_1}^{FT} > 0$ and $g_{c_2}^{FT} > 0$.

In either case, we find that

$$g_{c_1}^{FT} - g_{c_2}^{FT} = n_2 - n_1 > 0, \tag{49}$$

from which we obtain $g_{c_1}^{FT} > g_{c_2}^{FT}$.

In this case, we can consider Home and Foreign to be a developed and a developing country, respectively: (1) the population growth rate in developed countries is lower than that in developing countries; (2) per capita income growth in developed countries is higher than that in developing countries; and (3) developed countries are industrialized countries, while developing countries are agricultural countries.

[15] The necessary and sufficient condition for $g_{c_1}^{FT} > 0$ is given by $\frac{\beta-(1-\gamma)(1-\alpha)}{1-\alpha-\beta} n_1 + (1-\gamma)n_2 > 0$, which can be rewritten as $n_2 > \left[\frac{1-\alpha}{1-\alpha-\beta} - \frac{\beta}{(1-\gamma)(1-\alpha)}\right] n_1$. Note that the coefficient of n_1 is less than unity. Then, if $n_1 < n_2$, the condition $n_2 > \left[\frac{1-\alpha}{1-\alpha-\beta} - \frac{\beta}{(1-\gamma)(1-\alpha)}\right] n_1$ is always satisfied. Therefore, if $n_1 < n_2$, we necessarily have $g_{c_1}^{FT} > 0$.

Then, as stated above, when $\frac{1-\alpha}{1-\alpha-\beta}n_1 \leq n_2 \leq \frac{1}{1-\alpha-\beta}n_1$, the per capita consumption growth of the developing country is negative ($g_{c_2}^{FT} \leq 0$). To obtain positive growth, other things being equal, a developing country needs to lower its population growth and satisfy $n_2 < \frac{1-\alpha}{1-\alpha-\beta}n_1$. However, since population growth in developed countries is currently declining, that is, n_1 declines, even though some policies lower n_2, the above inequality will not be satisfied as long as n_1 decreases. Therefore, a decline in population growth in developing countries does not necessarily narrow the growth gap between them and developed countries.

These results are similar to those obtained by Sasaki (2011a). In his model, the trade pattern is fixed: the low-population-growth North produces only manufactured goods, whereas the high-population-growth South produces only agricultural goods. In our model, the trade patterns are endogenously determined, meaning that the country with low (high) population growth specializes in manufacturing (agriculture) in the long run.

Moreover, in Sasaki (2011a), if $n_1 < n_2$, we obtain $g_{c_1} > g_{c_2}$. In other words, the growth rate of per capita consumption in the low-population-growth manufacturing country is higher than that in the high-population-growth agricultural country, which is consistent with the results obtained in the present paper, that is, $g_{c_1}^{FT} > g_{c_2}^{FT}$ when $n_1 < n_2$.

These results are summarized in Table 1, which also compares the growth rates under autarky with those under free trade.[16]

[Table 1 around here]

Proposition 5. *Suppose that the population growth of Home is higher than that of Foreign. Under autarky, the per capita consumption growth of Home is higher than that of Foreign. Under free trade, given that Home diversifies and Foreign specializes in agriculture, the per capita consumption growth of Home is equal to that of Foreign.*

Proposition 6. *Suppose that the population growth of Home is lower than that of Foreign. Under autarky, the per capita consumption growth of Home is lower than that of Foreign. Under free trade, given that Home specializes in manufacturing and Foreign specializes in agriculture, the per capita consumption growth of Home is higher than that of Foreign.*

5 Conclusions

We built a two-country, two-sector, semi-endogenous growth model and investigated the relationship between trade patterns and per capita consumption growth, finding four interesting results.

[16] The Appendix also compares autarky with free trade.

First, Pattern 2 (i.e., Home diversifies and Foreign specializes in agriculture) is unsustainable if the population growth of Home is lower than that of Foreign. On the contrary, Pattern 3 (i.e., Home specializes in manufacturing and Foreign specializes in agriculture) is unsustainable if the population growth of Home is higher than that of Foreign.

Second, when the population growth of Home is higher than that of Foreign, the growth gap between Home and Foreign under autarky narrows and the growth rates of both countries are equalized by switching from autarky to free trade. In this case, Pattern 2 holds over time.

Third, when the population growth of Home is lower than that of Foreign, the growth gap between Home and Foreign under autarky may be reversed. In this case, Pattern 3 holds over time.

Finally, when the population growth of Home is higher than or equal to that of Foreign, each country's per capita consumption growth rate under free trade can be equal to or higher than its per capita consumption growth rate under autarky. By contrast, when the population growth of Home is lower than that of Foreign, by switching to free trade, Foreign may experience lower or even negative per capita consumption growth compared with under autarky.

In our model, learning-by-doing (i.e., the positive externality due to capital accumulation) determines total factor productivity (TFP). In turn, capital accumulation is ultimately determined by population growth. Hence, population growth ultimately determines growth in TFP. The structure of the comparative advantage of both countries at each point in time depends on the endowment of labor, endowment of capital, and TFP, which are supply-side factors, and the expenditure coefficients, which are demand-side factors. In the model, the factor that ultimately changes all these supply-side factors (labor endowment, capital endowment, TFP) is population growth and therefore population growth determines the comparative advantage of each country.

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Figures and tables

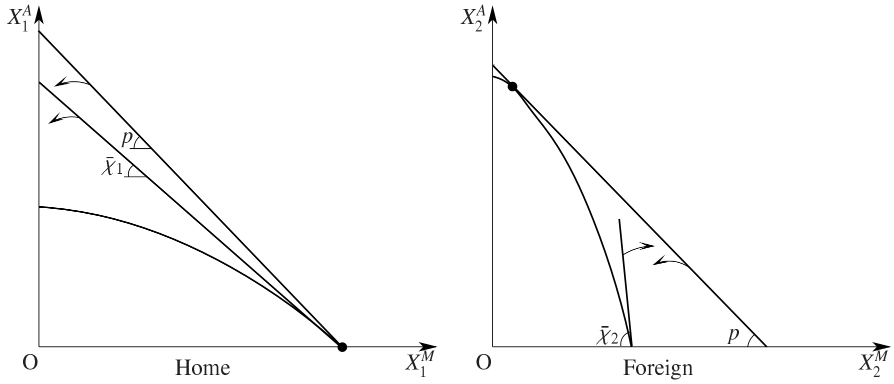

Figure 1: PPFs in Home and Foreign

Table 1: Comparison of autarky and free trade growth rates

Case	Case 1	Case 2	Case 3
Population growth	$n_1 = n_2$	$n_1 > n_2$	$n_1 < n_2$
Trade pattern*	2 and 3⋆	2	3
Relationship	$g_{c_1}^{AT} = g_{c_2}^{AT}$	$g_{c_1}^{AT} > g_{c_2}^{AT}$	$g_{c_1}^{AT} < g_{c_2}^{AT}$
between g_{c_1} and g_{c_2}	$g_{c_1}^{FT} = g_{c_2}^{FT}$	$g_{c_1}^{FT} = g_{c_2}^{FT}$	$g_{c_1}^{FT} > g_{c_2}^{FT}$
	$g_{c_1}^{AT} = g_{c_1}^{FT}$	$g_{c_1}^{AT} = g_{c_1}^{FT}$	$g_{c_1}^{AT} < g_{c_1}^{FT}$
	$g_{c_2}^{AT} = g_{c_2}^{FT}$	$g_{c_2}^{AT} < g_{c_2}^{FT}$	$g_{c_2}^{AT} > g_{c_2}^{FT}$
Negative growth◇	n/a	n/a	$g_{c_2}^{FT} < 0$ †

* Pattern 2 is the case where Home diversifies and Foreign asymptotically specializes completely in agriculture. Pattern 3 is the case where Home specializes completely in manufacturing and Foreign asymptotically specializes completely in agriculture.

⋆ If $\gamma \leq 1/2$, Pattern 2 is obtained, while if $1/2 < \gamma$, Pattern 3 is obtained.

◇ Depending on the conditions, negative growth is possible.

† $\frac{1-\alpha}{1-\alpha-\beta} n_1 < n_2 < \frac{1}{1-\alpha-\beta} n_1$.