Theoretical Analysis of the Effect of Fiscal Union: Keynesian Two-Country Model

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Abstract

In this paper, we investigate the effect of fiscal union related to a Capital Markets Union for the euro area in a Keynesian two-country model with a monetary union and imperfect capital mobility. We find that the increase in capital mobility between countries by creating a Capital Markets Union is a destabilizing factor, whereas an increase in fiscal transfers by creating a fiscal union is a stabilizing factor. Furthermore, we also find that an expansionary monetary policy implemented by the European Central Bank and an expansionary fiscal policy have positive effects on the real national income of both core and periphery countries.

Keyword: Keynesian Two-country model with monetary union, Economic stability, Fiscal union, Capital Markets Union, Post-Euro crisis

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1 Introduction

The Euro crisis of early 2010 was settled by the monetary policies of the European Central Bank, such as Long Term Refinancing Operations and Outright Monetary Transactions. However, the problem of a depression in the whole euro area and a gap between core countries and periphery countries persists in the post- Euro crisis era. It is important to adopt expansionary fiscal and monetary policies to solve the problem, but it is difficult to adopt an expansionary fiscal policy because periphery countries are required to obey fiscal discipline and implement austerity policies. Therefore, it is important to construct fiscal union with a system of fiscal transfers in the euro area.

In this situation, the European Commission proposed the concept of a Capital Markets Union in February 2015 in order to enlarge the European capital market, which is smaller than the market in the US. In other words, the European commission intends to increase private capital mobility in the euro area. However, it should be noted that increasing global capital mobility was one of the factors that contributed to the collapse of the Bretton Woods system. Furthermore, a private investment from core countries to periphery countries lead to a boom, like the housing bubble in that led to the global financial crisis and the Euro crisis. In this context, it is possible that a Capital Markets Union triggers more problems down the road. Therefore, it is important to consider an impact of a Capital Markets Union along with a fiscal union in the post-Euro crisis era for economic stability.

In this paper, we investigate the effect of a fiscal union in the Euro area related to Capital Markets Union in the post-Euro crisis era in a Keynesian two-country model with a monetary union and imperfect capital mobility. The Mundell-Fleming model developed by Mundell (1963) and Fleming (1962) is very useful for analyzing the international economy. The textbook Mundell-Fleming model is premised on the condition of a small country and perfect capital mobility. However, perfect capital mobility is unrealistic assumption for the world in general, and in the euro area. On this point, we analyze a fiscal union related to Capital Markets Union in the post-Euro crisis era using a Keynesian two-country model. Kawai (1994) explains the effect of monetary policy and fiscal policy with flexible and fixed exchange rates in a Keynesian two-country model; however, capital mobility is not included in that model. Asada (1997) describes an economy with both fixed and flexible exchange rates using a Keynesian model with small country and imperfect capital mobility. Asada (2016) analyzes an economy with a flexible exchange rate using a Keynesian model with large country and imperfect capital mobility with a three-dimensional system of nonlinear differential equations. Asada (2004) explains an economy with a fixed exchange rate using a Keynesian two-country model by a five-dimensional system of nonlinear differential equations. Asada, Chiarella, Flaschel, and Franke (2003) describe the international economy using a Keynes-Metzler-Goodwin dynamic model with a multi-dimensional system of nonlinear differential equations. Although there are many studies with two-country models as described above, few studies focus on the economy of the current euro area and the economy in a fiscal union using a two-country model. The main objective of this paper is an analysis of the effect of a fiscal union in the euro area related to a Capital Markets Union using a Keynesian two-country model with a monetary union and imperfect capital mobility consisting of a three-dimensional system of nonlinear differential equations based on Asada (2016). Based on this analysis, we suggest the adoption of a counter cyclical fiscal policy and the construction of a fiscal union in order to create a successful Capital Markets Union.

This paper is organized as follows. In Section 2, we formalize the model that consists of a three-dimensional system of nonlinear differential equations. In Section 3, we analyze the nature of the equilibrium solution of the model formulated in Section 2. In Section 4, we investigate conditions for the local stability of the equilibrium point. In Section 5, we formalize a model in the case of implementation of a fsical union. In Section 6, we analyze the nature of the equilibrium solution of the model formulated in Section 5. In Section 7, we consider the effect of monetary policy and fiscal policy in the case of a fiscal union using a comparative statics analysis. In Section 8, we conclude this paper.

2 Formulation of the Model: Current Euro Area Model

In this paper, we shall consider the economy in the euro area using a Keynesian two-country model with a monetary union and imperfect capital mobility. To analyze a monetary union like the euro area, we represent the exchange rate as follows.

$$E = E^e = \bar{E} = 1 \tag{1}$$

where E is the exchange rate and E^e is the expected exchange rate of the near future. We can assume that the exchange rate and the expected exchange rate is one since a single currency is used in a monetary union.

Under this assumption about the exchange rate, our model consists of the following system of dynamic equations in the short term.

(1) Behavioral equations

$$\dot{Y}_i = \alpha_i[C_i + I_i + G_i + J_i - Y_i] \ ; \ \alpha_i > 0, \tag{2}$$

$$C_i = c_i(Y_i - T_i) + C_{0i} \ ; \ 0 < c_i < 1, C_{0i} \geq 0, \tag{3}$$

$$T_i = \tau_i Y_i - T_{0i} \ ; \ 0 < \tau_i < 1, T_{0i} \geq 0, \tag{4}$$

$$I_i = I_i(r_i) \ ; \ I^i_{ri} = \frac{dI_i}{dr_i} < 0, \tag{5}$$

$$G_i = G_{0i} + \gamma_i(\bar{Y}_i - Y_i) \ ; \ \gamma_i > 0, \tag{6}$$

$$\frac{M_i}{p_i} = L_i(Y_i, r_i) \ ; \ \frac{\partial L_i}{\partial Y_i} > 0, L^i_{ri} = \frac{\partial L_i}{\partial r_i} < 0, \tag{7}$$

$$J_1 = J_1(Y_1, Y_2) \ ; \ J^1_{Y1} = \frac{\partial J_1}{\partial Y_1} < 0, J^1_{Y2} = \frac{\partial J_1}{\partial Y_2} > 0, \tag{8}$$

$$Q_1 = \beta\{r_1 - r_2\} \ ; \ \beta > 0, \tag{9}$$

(2) Definitional equations

$$A_1 = J_1 + Q_1, \tag{10}$$

$$p_1 J_1 + p_2 J_2 = 0, \tag{11}$$

$$p_1 Q_1 + p_2 Q_2 = 0, \tag{12}$$

$$p_1 A_1 + p_2 A_2 = 0, \tag{13}$$

$$\dot{M}_1 = p_1 A_1, \tag{14}$$

$$\bar{M} = M_1 + M_2, \tag{15}$$

where the subscript i ($i = 1, 2$) is the index number of a country, and the definitions of other symbols are as follows. Y_i is real national income, C_i is real private consumption expenditure, I_i is real private investment expenditure, G_i is real government expenditure, $\bar{Y}_i$ is the target real national income (this is not necessarily natural output), T_i is real income tax, T_{0i} is the negative income tax(basic income),

M_i is nominal money supply, p_i is price level, r_i is nominal rate of interest, J_i is real net export, Q_i is real capital account balance, A_i is real total balance of payments. The dot above the symbols represent derivatives with respect to time.

Eq. (2) is the disequilibrium quantity adjustment process in the goods market. The parameter α_i represents adjustment speed of goods market. Eq. (3) is the Keynesian consumption function indicating behavior of the consumer. Eq. (4) is the standard tax function. Eq. (5) is the standard Keynesian investment function. Eq. (6) is the government expenditure function. The parameter γ_i represent the degree of counter cyclical fiscal policy. The larger is γ_i, the larger is the counter cyclical government expenditure. Eq. (7) is the LM equation that represents the equilibrium condition in the monetary market. Eq. (8) is the real net export function of country 1. Eq. (9) is the real capital account balance function of country 1 in the model with imperfect capital mobility. The parameter β indicates the degree of mobility of international capital flows. The larger is β, the higher is the degree of mobility of international capital flows. The model of perfect capital mobility is a special case in which β is infinite, and the following equation is always established in a case of the fixed exchange rate system: $r_1 = r_2$. Eq. (10) is the definitional equation of the real total balance of payments of country 1. Eqs. (11), (12), and (13) imply that net export surplus, capital account balance surplus, and total balance of payments surplus of a country must be accompanied by the same amounts of the current account deficit, capital account balance deficit, and total balance of payments deficit of another country, respectively. Eq. (14) means that nominal money supply increases (decreases) according to the total balance of payment surplus (deficit) of country 1. Eq. (15) indicates that a total nominal money supply of two countries are fixed by the ECB.

Furthermore, we assume a fixed price economy.

$$p_1 = p_2 = 1 \tag{16}$$

In this paper, to simplify the analysis, we focus on a fixed price economy in the short run. This assumption eliminates price fluctuations. Therefore, we do not deal with the issues of inflation and deflation.

Fixing real government expenditure G_{0i}, marginal tax rate τ_i, and total nominal money supply of two countries $\bar{M}$ as policy parameters, the system of Eqs. (2)–(16) determines the dynamics of Y_i, C_i, G_i,

T_i, I_i, r_i, J_i, Q_i, A_i, M_i, and $p_i (i = 1, 2)$.

Then, we can transform this system into a more compact system. We obtain the following LM equation by solving Eq. (7) with respect to r_i.

$$r_i = r_i(Y_i, M_i) \; ; \; r^i_{Yi} = \frac{\partial r_i}{\partial Yi} = -\frac{L^i_{Y_i}}{L^i_{r_i}} > 0, r^i_{Mi} = \frac{\partial r_i}{\partial M_i} = \frac{1}{L^i_{r_i}} < 0. \tag{17}$$

Given policy parameters $\bar{M}$, G_i, and τ_i $(i = 1, 2)$, we can obtain the following three-dimensional system of nonlinear differential equations by substituting Eqs. (3), (4), (5), (6), (8), (11), (15), (16), and (17) into Eq. (2), and Eqs. (8), (9), (10), (15), (16), and (17) into Eq. (14).

$$\dot{Y}_1 = \alpha_1\{c_1(1-\tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\bar{Y}_1 - Y_1) + I_1(r_1(Y_1, M_1)) + J_1(Y_1, Y_2) - Y_1\}$$

$$= F_1(Y_1, Y_2, M_1; \alpha_1), \tag{18}$$

$$\dot{Y}_2 = \alpha_2\{c_2(1-\tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\bar{Y}_2 - Y_2) + I_2(r_2(Y_2, \bar{M} - M_1)) - J_1(Y_1, Y_2) - Y_2\}$$

$$= F_2(Y_1, Y_2, M_1; \alpha_2), \tag{19}$$

$$\dot{M}_1 = J_1(Y_1, Y_2) + \beta\{r_1(Y_1, M_1) - r_2(Y_2, \bar{M} - M_1)\} = F_3(Y_1, Y_2, M_1; \beta). \tag{20}$$

3 Nature of the Equilibrium Solution: Current Euro Area Model

The equilibrium solution (Y_1^*, Y_2^*, M_1^*) of the system Eqs. (18)–(20) is determined by the following system of equations.

$$c_1(1-\tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\bar{Y}_1 - Y_1) + I_1(r_1(Y_1, M_1)) + J_1(Y_1, Y_2) - Y_1 = 0, \tag{21}$$

$$c_2(1-\tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\bar{Y}_2 - Y_2) + I_2(r_2(Y_2, \bar{M} - M_1)) - J_1(Y_1, Y_2) - Y_2 = 0, \tag{22}$$

$$J_1(Y_1, Y_2) + \beta\{r_1(Y_1, M_1) - r_2(Y_2, \bar{M} - M_1)\} = 0. \tag{23}$$

Under the assumption that β is sufficiently large, we obtain the following equations by solving Eq. (23) with respect to M_1.

$$M_1 = \tilde{M}_1(Y_1, Y_2, \bar{M})\ ;\ \tilde{M}^1_{Y1} = \frac{\partial \tilde{M}_1}{\partial Y_1} = -(\underset{(-)}{J^1_{Y1}} + \beta \underset{(+)}{r^1_{Y1}})/\beta(\underset{(-)}{r^1_{M1}} + \underset{(-)}{r^2_{\bar{M}-M1}}) > 0, \tag{24}$$

$$\tilde{M}^1_{Y2} = \frac{\partial \tilde{M}_1}{\partial Y_2} = -(\underset{(+)}{J^1_{Y2}} - \beta \underset{(+)}{r^2_{Y2}})/\beta(\underset{(-)}{r^1_{M1}} + \underset{(-)}{r^2_{\bar{M}-M1}}) < 0,$$

$$\tilde{M}^1_{\bar{M}} = \frac{\partial \tilde{M}_1}{\partial \bar{M}} = \underset{(-)}{r^2_{\bar{M}-M1}}/(\underset{(-)}{r^1_{M1}} + \underset{(-)}{r^2_{\bar{M}-M1}}) > 0.$$

Then, we have the following equations by substituting Eq. (24) into Eqs. (21) and (22).

$$\begin{aligned} U_1(Y_1, Y_2) =& c_1(1-\tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\bar{Y_1} - Y_1) + I_1(r_1(Y_1, \tilde{M}_1(Y_1, Y_2, \bar{M}))) \\ & + J_1(Y_1, Y_2) - Y_1 \\ =& 0, \end{aligned} \tag{25}$$

$$\begin{aligned} U_2(Y_1, Y_2) =& c_2(1-\tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\bar{Y_2} - Y_2) + I_2(r_2(Y_2, \bar{M} - \tilde{M}_1(Y_1, Y_2, \bar{M}))) \\ & - J_1(Y_1, Y_2) - Y_2 \\ =& 0. \end{aligned} \tag{26}$$

Totally differentiating Eq. (25) and (26), we have the following relationships.

$$\left.\frac{dY_2}{dY_1}\right|_{U_1=0} = -\frac{U_{11}}{U_{12}} = [\underbrace{\{1 - c_1(1-\tau_1)\}}_{(+)} - \underset{(-)}{I^1_{r1}} \underset{(+)}{r^1_{Y1}} - \underset{(-)}{J^1_{Y1}} + \gamma_1]/\underset{(+)}{J^1_{Y2}} > 0, \tag{27}$$

$$\left.\frac{dY_2}{dY_1}\right|_{U_2=0} = -\frac{U_{21}}{U_{22}} = \underset{(-)}{J^1_{Y1}}/[\underbrace{\{1 - c_2(1-\tau_2)\}}_{(+)} - \underset{(-)}{I^2_{r2}} \underset{(+)}{r^2_{Y2}} + \underset{(+)}{J^1_{Y2}} + \gamma_2] < 0, \tag{28}$$

where $U_{11} = \partial U_1/\partial Y_1 = -\{1 - c_1(1-\tau_1)\} + I^1_{r1}r^1_{Y1} + J^1_{Y1} - \gamma_1$, $U_{12} = \partial U_1/\partial Y_2 = J^1_{Y2}$, $U_{21} = \partial U_2/\partial Y_1 = J^2_{Y1}$, $U_{22} = \partial U_2/\partial Y_2 = -\{1 - c_2(1-\tau_2)\} + I^2_{r2}r^2_{Y2} + J^1_{Y2} - \gamma_2$. Using Eq. (27) and (28), we can obtain equilibrium national incomes (Y_1^*, Y_2^*) from the point on the plane (Y_1, Y_2) at the intersection of $U_1(Y_1, Y_2) = 0$ with $U_2(Y_1, Y_2) = 0$, given the fiscal policy parameters $(G_{01}, \tau_1, \gamma_1, G_{02}, \tau_2, and \gamma_2)$, and monetary policy parameter $(\bar{M})$. Further, substituting (Y_1^*, Y_2^*) into Eq. (23), we have the equilibrium money supply of country 1 (M_1^*).

4 Local Stability Analysis: Current Euro Area Model

In this section, we shall assume that there exists a unique equilibrium solution $(Y_1^*, Y_2^*, M_1^*) > (0,0,0)$ and analyze the local stability of this equilibrium solution. We can write the Jacobian matrix of the system of Eqs. (18)–(20) that are evaluated at the equilibrium point as follows.

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} = \begin{bmatrix} \alpha_1 G_{11} & \alpha_1 G_{12} & \alpha_1 G_{13} \\ \alpha_2 G_{21} & \alpha_2 G_{22} & \alpha_2 G_{23} \\ F_{31}(\beta) & F_{32}(\beta) & F_{33}(\beta) \end{bmatrix}, \tag{29}$$

where

$$G_{11} = -\underbrace{\{1 - c_1(1-\tau_1)\}}_{(+)} + \underset{(-)}{I^1_{r1}} \underset{(+)}{r^1_{Y1}} + \underset{(-)}{J^1_{Y1}} - \gamma_1 < 0,\ G_{12} = \underset{(+)}{J^1_{Y2}} > 0,\ G_{13} = \underset{(-)}{I^1_{r1}} \underset{(-)}{r^1_{M1}} > 0,$$

$$G_{21} = -\underset{(-)}{J^1_{Y1}} > 0,\ G_{22} = -\underbrace{\{1 - c_2(1-\tau_2)\}}_{(+)} + \underset{(-)}{I^2_{r2}} \underset{(+)}{r^2_{Y2}} - \underset{(+)}{J^1_{Y2}} - \gamma_2 < 0,\ G_{23} = -\underset{(-)}{I^2_{r2}} \underset{(-)}{r^2_{\bar{M}-M1}} < 0,$$

$$F_{31}(\beta) = \underset{(-)}{J^1_{Y1}} + \beta \underset{(+)}{r^1_{Y1}},\ F_{32}(\beta) = \underset{(+)}{J^1_{Y2}} - \beta \underset{(+)}{r^2_{Y2}},\ F_{33}(\beta) = \beta(\underset{(-)}{r^1_{M1}} + \underset{(-)}{r^2_{\bar{M}-M1}}) < 0.$$

We can express the characteristic equation of this system as

$$f(\lambda) = |\lambda I - J| = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{30}$$

where

$$a_1 = -traceJ = -\alpha_1 \underset{(-)}{G_{11}} - \alpha_2 \underset{(-)}{G_{22}} - \underset{(-)}{F_{33}(\beta)} = a_1(\beta) > 0, \tag{31}$$

$$\begin{aligned} a_2 &= \alpha_1\alpha_2 \begin{vmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{vmatrix} + \alpha_1 \begin{vmatrix} G_{11} & G_{13} \\ F_{31}(\beta) & F_{33}(\beta) \end{vmatrix} + \alpha_2 \begin{vmatrix} G_{22} & G_{23} \\ F_{32}(\beta) & F_{33}(\beta) \end{vmatrix} \\ &= \alpha_1\alpha_2(\underset{(-)}{G_{11}}\underset{(-)}{G_{22}} - \underset{(+)}{G_{12}}\underset{(+)}{G_{21}}) + \alpha_1(\underset{(-)}{G_{11}}\underset{(-)}{F_{33}(\beta)} - \underset{(+)}{G_{13}}\underset{(?)}{F_{31}(\beta)}) + \alpha_2(\underset{(-)}{G_{22}}\underset{(-)}{F_{33}(\beta)} - \underset{(-)}{G_{23}}\underset{(?)}{F_{32}(\beta)}) \\ &= a_2(\beta), \end{aligned} \tag{32}$$

$$
\begin{aligned}
a_3 &= -\det J \\
&= -\begin{vmatrix} \alpha_1 G_{11} & \alpha_1 G_{12} & \alpha_1 G_{13} \\ \alpha_2 G_{21} & \alpha_2 G_{22} & \alpha_2 G_{23} \\ F_{31}(\beta) & F_{32}(\beta) & F_{33}(\beta) \end{vmatrix} \\
&= \alpha_1\alpha_2[-\underset{(-)}{G_{11}}\underset{(-)}{G_{22}}\underset{(-)}{F_{33}(\beta)} + \underset{(-)}{G_{11}}\underset{(?)}{F_{32}(\beta)}\underset{(-)}{G_{23}} - \underset{(+)}{G_{12}}\underset{(-)}{G_{23}}\underset{(?)}{F_{31}(\beta)} + \underset{(+)}{G_{12}}\underset{(+)}{G_{21}}\underset{(-)}{F_{33}(\beta)} - \underset{(+)}{G_{13}}\underset{(?)}{F_{32}(\beta)}\underset{(+)}{G_{21}} \\
&\quad + \underset{(+)}{G_{13}}\underset{(-)}{G_{22}}\underset{(?)}{F_{31}(\beta)}] \\
&= a_3(\beta),
\end{aligned}
\tag{33}
$$

$$
a_1a_2 - a_3 = a_1(\beta)a_2(\beta) - a_3(\beta) = B\beta^2 + C\beta + D \ ; \tag{34}
$$

$$
B = -\underset{(-)}{F_{33}(\beta)}\{\alpha_1(\underset{(-)}{G_{11}}\underset{(-)}{F_{33}(\beta)} - \underset{(+)}{G_{13}}\underset{(?)}{F_{31}(\beta)}) + \alpha_2(\underset{(-)}{G_{22}}\underset{(-)}{F_{33}(\beta)} - \underset{(-)}{G_{23}}\underset{(?)}{F_{32}(\beta)})\}.
$$

All of the roots of the characteristic equation (30) have negative real parts if and if only the following Routh-Hurwitz conditions are satisfied[1)].

$$
a_1 > 0,\ a_2 > 0,\ a_3 > 0,\ a_1a_2 - a_3 > 0. \tag{35}
$$

Then, the equilibrium point of the system (18)–(20) is locally stable.

Proposition 1.

(i) Suppose that the parameter β is fixed at any level. Then, the equilibrium point of the system (18)–(20) is locally stable if the parameter γ_1 and γ_2 is sufficiently large.

(ii) Suppose that the parameter γ_1 and γ_2 is fixed at any level. Then, the equilibrium point of the system (18)–(20) is locally unstable if the parameter β is sufficiently large.

Proof.

(i) Suppose that the parameter β is fixed at any level. We can express Eq. (32) as follows.

$$
a_2 = a\gamma_1\gamma_2 + b\gamma_1 + c\gamma_2 + d. \tag{36}
$$

1) Gandolfo(2009) pp.229-240.

Then, a is positive because $\alpha_1\alpha_2 > 0$ holds. Thus, we have $a_2 > 0$, if the parameter γ_1 and γ_2 is sufficiently large. For the same reason, we can obtain $a_3 > 0$, $a_1a_2 - a_3 > 0$, if the parameter γ_1 and γ_2 is sufficiently large. Therefore, the equilibrium point of the system (18)–(20) is locally stable because the Routh-Hurwitz conditions are satisfied.

(ii) Suppose that the parameter γ_1 and γ_2 is fixed at any level. If the parameter β is sufficiently large, we have $F_{31}(\beta) > 0$, $F_{32}(\beta) < 0$. a_2 is written as follow.

$$a_2(\beta) = \alpha_1\alpha_2(\underset{(-)}{G_{11}}\underset{(-)}{G_{22}} - \underset{(+)}{G_{12}}\underset{(+)}{G_{21}}) + \alpha_1(\underset{(-)}{G_{11}} - \underset{(+)}{G_{13}}\underset{(+)}{F_{31}(\beta)}) + \alpha_2(\underset{(-)}{G_{22}} - \underset{(-)}{G_{23}}\underset{(-)}{F_{32}(\beta)}) \tag{37}$$

Then, we have $a_2 < 0$ if the parameter β is sufficiently large. Thus, the equilibrium point of the system (18)–(20) is locally unstable because the Routh-Hurwitz conditions are not satisfied. □

Proposition 1 indicates an increase in capital mobility in a monetary union lead to instability. Asada (1997) describes economically the effect of the parameter β on an economic stability using a Keynesian model with small country. In a similar way, Proposition 1 can be interpreted economically as follows.

First, suppose that the parameter β is sufficiently large. If real national income of country 1 Y_1 falls below the equilibrium point by an exogenous shock, a decrease in Y_1 leads to a decrease in nominal rate of interest r_1 and leads to a small r_1 relative to r_2. An increase in capital transfer to country 2 from country 1 brings about a deficit of capital account balance Q_1 and a decrease in money supply M_1. If the parameter β is large, this 'capital account effect' is relatively strong compared with the counteracting 'current account effect'. However, a decrease in M_1 leads to a rapid increase in r_1. An increase in r_1 restrains real private investment expenditure I_1. Then, a decrease in an investment in country 1 leads to a further decrease in Y_1. On the other hand, a decrease in r_1 brings about an increase in Y_1. However, the former effect on Y_1 is larger than the latter if the parameter β is sufficiently large. Thus, a large β is a destabilizing factor in monetary union.

Second, suppose that the parameter β is sufficiently small. If Y_1 falls below the equilibrium point by an exogenous shock, a decrease in Y_1 leads to a decrease in r_1. A decrease in r_1 brings about an increase in Y_1. On the other hand, a decrease in Y_1 leads to an increase in real net export J_1 through a decrease in import. An increase in J_1 brings about an increase in M_1. If the parameter β is small, this 'current account effect' is relatively strong compared with the counteracting 'capital account effect'. Then, an increase in M_1 lead to an increase in I_1 and Y_1 through a decrease in r_1 if the parameter β is sufficiently small. Thus, a small β is a stabilizing factor in monetary union.

It is important to note that these mechanisms work under fixed exchange rate system. In case of the flexible exchange rate system, a large β may have the stabilizing effect [2].

It is necessary to make the parameter γ_i large if policymakers intend to increase capital mobility in the area, while maintaining stability. This result is contrary to that in Ingram (1973), which stresses the increase of the degree of financial integration as a condition of creating an optimum currency area.

An integration of currencies presuppose a liberalization of capital mobility in the area. However, free capital mobility for private market participants destabilizes economy in a monetary union in the absence of the support provided by government expenditure. Capital is not as mobile in the euro area as compared to the US, because there are barriers of history, culture, and regulation in the euro area. To improve these problems, the European Commission proposed the creation of a Capital Markets Union in February 2015. The European Commission intends to construct a Capital Markets Union according the following principles [3].

1. it should maximize the benefits of capital markets for the economy, job creation, and growth;
2. it should create a single market for capital for all 28 Member States by removing barriers to cross-border investment within the EU and fostering stronger connections with global capital markets;
3. it should be built on firm foundations of financial stability, with a single rulebook for financial services which is effectively and consistently enforced;
4. it should ensure an effective level of consumer and investor protection; and
5. it should help to attract investment from all over the world and increase EU competitiveness.

However, a Capital Markets Union will lead to an increase in the parameter β. The parameter γ_i is small in the euro area because each country in the euro area tends to adopt fiscal austerity policies. In this state, creating a Capital Markets Union is likely to increase instability in the euro area. Thus,

[2] See Asada (1997) and Asada (2016).
[3] European Commission (2015) p.5.

if the European Commission creates a Capital Markets Union, each country in the euro area must not adopt fiscal austerity policies but adopt counter-cyclical expansionary fiscal policy. However, it is difficult to adopt expansionary fiscal policy in each country independently, because periphery countries such as Greece are required to obey fiscal discipline. To solve this problem, it is important to create a fiscal union.

5 Formulation of the Model: Fiscal Union Model

In this section, we analyze the stability of the equilibrium point in the case of the creation of a fiscal union. The fiscal union should have a mechanism that triggers fiscal transfers from countries experiencing expansions to countries experiencing depressions. Kenen (1969) propose fiscal transfers for a condition of optimum currency area. Furthermore, De Grauwe (2016) present two points about the effect of the fiscal union [4]. First, the fiscal union creates an insurance mechanism triggering income transfers from countries experiencing good times to the countries hit by bad luck. In doing so, it reduces the pain in the countries hit by negative shock. Second, a fiscal union allows consolidation of a significant part of the debts of national government, thereby, protecting its members from liquidity crises and forced defaults.

In this paper, to focus on fiscal transfers as an insurance mechanism in a fiscal union, we add Equations about the fiscal transfer mechanism to Eq. (6) and formulate the system as follows.

$$G_1 = G_{01} + \gamma_1(\bar{Y_1} - Y_1) + \mu(\bar{Y_1} - Y_1)\ ;\ \gamma_1 > 0\ ;\ \mu > 0, \tag{38}$$

$$G_2 = G_{02} + \gamma_2(\bar{Y_2} - Y_2) - \mu(\bar{Y_1} - Y_1)\ ;\ \gamma_2 > 0\ ;\ \mu > 0, \tag{39}$$

where the parameter μ is the degree of the fiscal transfer. We assume a lopsided fiscal transfer to periphery countries from the core countries. In other words, these equations indicate that country 2 (core country) increases government expenditure to make transfers to country 1 (periphery country) in the event of a depression country 1 experiencing depression ($\bar{Y_1} > Y_1$), while country 1 does not make transfers to country 2 in the event of depression in country 2.

By adopting an approach similar to the one adopted for developing the expression for (18)–(20), we

[4] De Grauwe (2016) p.17.

formulate the systems as follows.

$$\dot{Y}_1 = \alpha_1\{c_1(1-\tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\bar{Y}_1 - Y_1) + \mu(\bar{Y}_1 - Y_1) + I_1(r_1(Y_1, M_1))$$
$$+ J_1(Y_1, Y_2) - Y_1\}$$
$$= F_1(Y_1, Y_2, M_1; \alpha_1), \tag{40}$$
$$\dot{Y}_2 = \alpha_2\{c_2(1-\tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\bar{Y}_2 - Y_2) - \mu(\bar{Y}_2 - Y_2) + I_2(r_2(Y_2, \bar{M} - M_1))$$
$$- J_1(Y_1, Y_2) - Y_2\}$$
$$= F_2(Y_1, Y_2, M_1; \alpha_2), \tag{41}$$
$$\dot{M}_1 = J_1(Y_1, Y_2) + \beta\{r_1(Y_1, M_1) - r_2(Y_2, \bar{M} - M_1)\} = F_3(Y_1, Y_2, M_1; \beta). \tag{42}$$

We investigate a nature of the equilibrium solution (Y_1^*, Y_2^*, M_1^*) that satisfies $\dot{Y}_1 = \dot{Y}_2 = \dot{M}_1 = 0$. The equilibrium solution of Eqs. (40)-(42) is determined as a solution of the following simultaneous equations.

$$c_1(1-\tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\bar{Y}_1 - Y_1) + \mu(\bar{Y}_1 - Y_1) + I_1(r_1(Y_1, M_1))$$
$$+ J_1(Y_1, Y_2) - Y_1 = 0, \tag{43}$$
$$c_2(1-\tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\bar{Y}_2 - Y_2) - \mu(\bar{Y}_1 - Y_1) + I_2(r_2(Y_2, \bar{M} - M_1))$$
$$- J_1(Y_1, Y_2) - Y_2 = 0, \tag{44}$$
$$J_1(Y_1, Y_2) + \beta\{r_1(Y_1, M_1) - r_2(Y_2, \bar{M} - M_1)\} = 0. \tag{45}$$

However, we leave out developing the equilibrium solution, because we can obtain the equilibrium solution in the case of a fiscal union using an approach similar to that used to obtain the equilibrium solution (Y_1^*, Y_2^*, M_1^*) in Section 3.

6 Local Stability Analysis: Fiscal Union Model

In this section, we shall assume that there exists a unique equilibrium solution $(Y_1^*, Y_2^*, M_1^*) > (0, 0, 0)$ in the case of a fiscal union and analyze the local stability of this equilibrium solution. We can write the

Jacobian matrix of the system of Eqs.(40)–(42) that are evaluated at the equilibrium point as follows.

$$J = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} = \begin{bmatrix} \alpha_1 K_{11} & \alpha_1 K_{12} & \alpha_1 K_{13} \\ \alpha_2 K_{21} & \alpha_2 K_{22} & \alpha_2 K_{23} \\ H_{31}(\beta) & H_{32}(\beta) & H_{33}(\beta) \end{bmatrix}, \tag{46}$$

where

$$K_{11} = -\underbrace{\{1 - c_1(1-\tau_1)\}}_{(+)} + \underset{(-)}{I^1_{r1}}\underset{(+)}{r^1_{Y1}} + \underset{(-)}{J^1_{Y1}} - \gamma_1 - \mu < 0,\ K_{12} = \underset{(+)}{J^1_{Y2}} > 0,\ K_{13} = \underset{(-)}{I^1_{r1}}\underset{(-)}{r^1_{M1}} > 0,$$

$$K_{21} = \mu - \underset{(-)}{J^1_{Y1}} > 0,\ K_{22} = -\underbrace{\{1 - c_2(1-\tau_2)\}}_{(+)} + \underset{(-)}{I^2_{r2}}\underset{(+)}{r^2_{Y2}} - \underset{(+)}{J^1_{Y2}} - \gamma_2 < 0,\ K_{23} = -\underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M_1}} < 0,$$

$$H_{31}(\beta) = \underset{(-)}{J^1_{Y1}} + \beta \underset{(+)}{r^1_{Y1}},\ H_{32}(\beta) = \underset{(+)}{J^1_{Y2}} - \beta \underset{(+)}{r^2_{Y2}},\ H_{33}(\beta) = \beta(\underset{(-)}{r^1_{M1}} + \underset{(-)}{r^2_{\bar{M}-M1}}) < 0.$$

We can express the characteristic equation of this system as

$$f(\lambda) = |\lambda I - J| = \lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \tag{47}$$

where

$$b_1 = -traceJ = -\alpha_1 \underset{(-)}{K_{11}} - \alpha_2 \underset{(-)}{K_{22}} - \underset{(-)}{H_{33}(\beta)} = b_1(\beta) > 0, \tag{48}$$

$$b_2 = \alpha_1\alpha_2 \begin{vmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{vmatrix} + \alpha_1 \begin{vmatrix} K_{11} & K_{13} \\ H_{31}(\beta) & H_{33}(\beta) \end{vmatrix} + \alpha_2 \begin{vmatrix} K_{22} & K_{23} \\ H_{32}(\beta) & H_{33}(\beta) \end{vmatrix}$$

$$= \alpha_1\alpha_2(\underset{(-)}{K_{11}}\underset{(-)}{K_{22}} - \underset{(+)}{K_{12}}\underset{(+)}{K_{21}}) + \alpha_1(\underset{(-)}{K_{11}}\underset{(-)}{H_{33}(\beta)} - \underset{(+)}{K_{13}}\underset{(?)}{H_{31}(\beta)}) + \alpha_2(\underset{(-)}{K_{22}}\underset{(-)}{H_{33}(\beta)} - \underset{(-)}{K_{23}}\underset{(?)}{H_{32}(\beta)})$$

$$= b_2(\beta), \tag{49}$$

$$b_3 = -\det J$$

$$= -\begin{vmatrix} \alpha_1 K_{11} & \alpha_1 K_{12} & \alpha_1 K_{13} \\ \alpha_2 K_{21} & \alpha_2 K_{22} & \alpha_2 K_{23} \\ H_{31}(\beta) & H_{32}(\beta) & H_{33}(\beta) \end{vmatrix}$$

$$
\begin{aligned}
&= \alpha_1\alpha_2[-\underset{(-)}{K_{11}}\underset{(-)}{K_{22}}\underset{(-)}{H_{33}(\beta)} + \underset{(-)}{K_{11}}\underset{(?)}{H_{32}(\beta)}\underset{(-)}{K_{23}} - \underset{(+)}{K_{12}}\underset{(-)}{K_{23}}\underset{(?)}{H_{31}(\beta)} + \underset{(+)}{K_{12}}\underset{(+)}{K_{21}}\underset{(-)}{H_{33}(\beta)} - \underset{(+)}{K_{13}}\underset{(?)}{H_{32}(\beta)}\underset{(+)}{K_{21}} \\
&\quad + \underset{(+)}{K_{13}}\underset{(-)}{K_{22}}\underset{(?)}{H_{31}(\beta)}] \\
&= b_3(\beta), \qquad (50)
\end{aligned}
$$

$$
b_1b_2 - b_3 = b_1(\beta)b_2(\beta) - b_3(\beta) = B\beta^2 + C\beta + D \;;
$$

$$
B = -\underset{(-)}{H_{33}(\beta)}[\alpha_1(\underset{(-)}{K_{11}}\underset{(-)}{H_{33}(\beta)} - \underset{(+)}{K_{13}}\underset{(?)}{H_{31}(\beta)}) + \alpha_2(\underset{(-)}{K_{22}}\underset{(-)}{H_{33}(\beta)} - \underset{(-)}{K_{23}}\underset{(?)}{H_{32}(\beta)})]. \qquad (51)
$$

All of the roots of the characteristic equation (47) have negative real parts if and if only the following Routh-Hurwitz conditions are satisfied.

$$
b_1 > 0,\ b_2 > 0,\ b_3 > 0,\ b_1b_2 - b_3 > 0. \qquad (52)
$$

Then, the equilibrium point of the system (40)-(42) is locally stable.

Proposition 2.

(i) Suppose that the parameter β is fixed at any level. Then, the equilibrium point of the system (40)-(42) is locally stable if the parameters γ_1 and γ_2 or μ are sufficiently large.

(ii) Suppose that the parameter γ_1, γ_2, and μ are fixed at any level. Then, the equilibrium point of the system (40)-(42) are locally unstable if the parameter β is sufficiently large.

Proof.

(i) Suppose that the parameter β is fixed at any level. If the parameter μ is constant, we can express Eq. (49) as follows.

$$
b_2 = a\gamma_1\gamma_2 + b\gamma_1 + c\gamma_2 + d. \qquad (53)
$$

Then, a is positive because $\alpha_1\alpha_2 > 0$ holds. If the parameter γ_1 and γ_2 is constant, b_2 is a quadratic function with respect to μ. The coefficient of the quadratic term is positive. Thus, we have $b_2 > 0$, if the parameter μ is sufficiently large. To summarize, we have $b_2 > 0$, if the parameters γ_1 and γ_2 or μ are sufficiently large. For the same reason, we can obtain $b_3 > 0$, $b_1b_2 - b_3 > 0$, if the parameters γ_1 and

γ_2 or μ are sufficiently large. Therefore, the equilibrium point of the system (40)–(42) is locally stable because the Routh-Hurwitz conditions are satisfied.

(ii) Suppose that the parameters γ_1, γ_2, and μ are fixed at any level. If the parameter β is sufficiently large, we have $H_{31}(\beta) > 0$, $H_{32}(\beta) < 0$. b_2 is written as follows.

$$b_2(\beta) = \alpha_1\alpha_2(\underset{(-)}{K_{11}}\underset{(-)}{K_{22}} - \underset{(+)}{K_{12}}\underset{(+)}{K_{21}}) + \alpha_1(\underset{(-)}{K_{11}} - \underset{(+)}{K_{13}}\underset{(+)}{H_{31}(\beta)}) + \alpha_2(\underset{(-)}{K_{22}} - \underset{(-)}{K_{23}}\underset{(-)}{H_{32}(\beta)}). \tag{54}$$

Then, we have $b_2 < 0$ if the parameter β is sufficiently large. Thus, the equilibrium point of the system (40)–(42) is locally unstable because the Routh-Hurwitz conditions are not satisfied. □

Proposition 2 indicates that it is necessary to adopt a counter-cyclical fiscal policy and a fiscal transfer mechanism in a fiscal union, like in a Capital Markets Union, as is indicated in Proposition 1. However, the parameter γ_i is small, because the countries in the euro area tend to adopt austerity fiscal policies. Furthermore, the stability effect of the parameter μ does not exist due to the absence of a fiscal union.

7 Comparative Statics Analysis of Fiscal and Monetary Policy: Fiscal Union Model

In this section, we shall consider the effect of monetary policy and fiscal policy while maintaining economic stability by creating a fiscal union.

7.1 Comparative Statics Analysis of Monetary Policy

In Eqs. (43)–(45), we investigate the impact of the endogenous variables Y_1, Y_2 and M_1 on the equilibrium value if the total monetary supply of both countries $\bar{M}$ changes by the amount $d\bar{M}$. Totally differentiating Eqs. (43)–(45), we have the following equations.

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} dY_1 \\ dY_2 \\ dM_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}}d\bar{M} \\ -\beta \underset{(-)}{r^2_{\bar{M}-M1}}d\bar{M} \end{bmatrix}, \tag{55}$$

where

$$S_{11} = -\underbrace{\{1 - c_1(1-\tau_1)\}}_{(+)} + \underset{(-)}{I^1_{r1}}\underset{(+)}{r^1_{Y1}} + \underset{(-)}{J^1_{Y1}} - \gamma_1 - \mu < 0,\ S_{12} = \underset{(+)}{J^1_{Y2}} > 0,\ S_{13} = \underset{(-)}{I^1_{r1}}\underset{(-)}{r^1_{M1}} > 0,$$

$$S_{21} = \mu - \underset{(-)}{J^1_{Y1}} > 0,\ S_{22} = -\underbrace{\{1 - c_2(1-\tau_2)\}}_{(+)} + \underset{(-)}{I^2_{r2}}\underset{(+)}{r^2_{Y2}} - \underset{(+)}{J^1_{Y2}} - \gamma_2 < 0,\ S_{23} = -\underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M_1}} < 0,$$

$$S_{31}(\beta) = \underset{(-)}{J^1_{Y1}} + \beta \underset{(+)}{r^1_{Y1}} < 0,\ S_{32}(\beta) = \underset{(+)}{J^1_{Y2}} - \beta \underset{(+)}{r^2_{Y2}} > 0,\ S_{33}(\beta) = \beta(\underset{(-)}{r^1_{M1}} + \underset{(-)}{r^2_{\bar{M}-M1}}) < 0.$$

Expressing a coefficient matrix of the left hand side of Eq. (55) as S, we have the following $\det S$.

$$\det S = \underset{(-)}{S_{11}}\underset{(-)}{S_{22}}\underset{(-)}{S_{33}} - \underset{(-)}{S_{11}}\underset{(+)}{S_{32}}\underset{(-)}{S_{23}} + \underset{(+)}{S_{12}}\underset{(-)}{S_{23}}\underset{(-)}{S_{31}} - \underset{(+)}{S_{12}}\underset{(+)}{S_{21}}\underset{(-)}{S_{33}} + \underset{(+)}{S_{13}}\underset{(+)}{S_{32}}\underset{(+)}{S_{21}} - \underset{(+)}{S_{13}}\underset{(-)}{S_{22}}\underset{(-)}{S_{31}}. \tag{56}$$

Now, let us make the following assumption.

Assumption 1. S_{11} and S_{22} are so sufficiently large that we have $\det S < 0$.

The inequality indicated in Assumption 1 is satisfied if the absolute value of S_{11} and S_{22} is relatively larger than the absolute value of the other terms. In this assumption, we investigate the change of an endogenous variable in (55) using a Cramer's rule.

$$\frac{dY_1}{d\bar{M}} = (Y^1_{\bar{M}})^* = \frac{1}{\det S}\begin{vmatrix} 0 & \underset{(+)}{S_{12}} & \underset{(+)}{S_{13}} \\ -\underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}} & \underset{(-)}{S_{22}} & \underset{(-)}{S_{23}} \\ -\beta\underset{(-)}{r^2_{\bar{M}-M1}} & \underset{(+)}{S_{32}} & \underset{(-)}{S_{33}} \end{vmatrix}$$

$$= \frac{1}{\det S}\{-\beta\underset{(-)}{r^2_{\bar{M}-M1}}(\underset{(+)}{S_{12}}\underset{(-)}{S_{23}} - \underset{(+)}{S_{13}}\underset{(-)}{S_{22}}) - \underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}}(\underset{(+)}{S_{13}}\underset{(+)}{S_{32}} - \underset{(+)}{S_{12}}\underset{(-)}{S_{33}})\} > 0, \tag{57}$$

$$\frac{dY_2}{d\bar{M}} = (Y^2_{\bar{M}})^* = \frac{1}{\det S}\begin{vmatrix} \underset{(-)}{S_{11}} & 0 & \underset{(+)}{S_{13}} \\ \underset{(-)}{S_{22}} & -\underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}} & \underset{(-)}{S_{23}} \\ \underset{(-)}{S_{31}} & -\beta\underset{(-)}{r^2_{\bar{M}-M1}} & \underset{(-)}{S_{33}} \end{vmatrix}$$

$$= \frac{1}{\det S}\{-\beta\underset{(-)}{r^2_{\bar{M}-M1}}(\underset{(+)}{S_{13}}\underset{(+)}{S_{21}} - \underset{(-)}{S_{11}}\underset{(-)}{S_{23}}) - \underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}}(\underset{(-)}{S_{11}}\underset{(-)}{S_{33}} - \underset{(+)}{S_{13}}\underset{(-)}{S_{31}})\} > 0, \tag{58}$$

$$\frac{dM_1}{d\bar{M}} = (M^1_{\bar{M}})^* = \frac{1}{\det S} \begin{vmatrix} \underset{(-)}{S_{11}} & \underset{(+)}{S_{12}} & 0 \\ \underset{(-)}{S_{22}} & \underset{(-)}{S_{22}} & -\underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}} \\ \underset{(-)}{S_{31}} & \underset{(+)}{S_{32}} & -\beta \underset{(-)}{r^2_{\bar{M}-M1}} \end{vmatrix}$$

$$= \frac{1}{\det S}\{-\beta \underset{(-)}{r^2_{\bar{M}-M1}}(\underset{(-)}{S_{11}}\underset{(-)}{S_{22}} - \underset{(+)}{S_{12}}\underset{(+)}{S_{21}}) - \underset{(-)}{I^2_{r2}}\underset{(-)}{r^2_{\bar{M}-M1}}(\underset{(+)}{S_{12}}\underset{(-)}{S_{31}} - \underset{(-)}{S_{11}}\underset{(+)}{S_{32}})\} > 0. \tag{59}$$

Then, we can establish the following proposition.

Proposition 3. If the parameters γ_i or μ are sufficiently large so that the absolute values of S_{11} and S_{22} as a stabilizing factors of the system are larger than the absolute values of the other terms, we have the following relationships.

$$\frac{dY_1}{d\bar{M}} > 0, \ \frac{dY_2}{d\bar{M}} > 0, \ \frac{dM_1}{d\bar{M}} > 0. \tag{60}$$

Proposition 3 implies that the monetary policy of a supranational central bank like the ECB can have an impact for an economy in the area if a some additional assumptions are satisfied in the two-country model with a monetary union and imperfect capital mobility. Therefore, a quantitative easing implemented by the ECB has a positive impact for the euro are economy [5].

7.2 Comparative Statics Analysis of Fiscal Policy

We develop a comparative statics analysis of fiscal policy under Assumption 1. In Eqs. (43)–(45), we investigate the impact of the endogenous variables Y_1, Y_2 and M_1 on the equilibrium value if the fiscal policy parameters τ_i and G_{0i} change by the amount $d\tau_i$ and dG_{0i}, respectively. Totally differentiating Eqs. (43)–(45) in the case of $i = 1$, we have the following equations.

$$\begin{bmatrix} \underset{(-)}{S_{11}} & \underset{(+)}{S_{12}} & \underset{(+)}{S_{13}} \\ \underset{(+)}{S_{21}} & \underset{(-)}{S_{22}} & \underset{(-)}{S_{23}} \\ \underset{(-)}{S_{31}} & \underset{(+)}{S_{32}} & \underset{(-)}{S_{33}} \end{bmatrix} \begin{bmatrix} dY_1 \\ dY_2 \\ dM_1 \end{bmatrix} = \begin{bmatrix} c_1 Y_1 d\tau_1 - dG_{01} \\ 0 \\ 0 \end{bmatrix}. \tag{61}$$

[5] For the details of expansionary monetary policies of the ECB, refer to Nakao (2016).

First, we consider the case when $d\tau_1 = 0$ and $dG_{01} \neq 0$. We investigate the change of endogenous variables in (55) using a Cramer's rule.

$$\frac{dY_1}{dG_{01}} = (Y^1_{G_{01}})^* = \frac{1}{\det S}\begin{vmatrix} -1 & \underset{(+)}{S_{12}} & \underset{(+)}{S_{13}} \\ 0 & \underset{(-)}{S_{22}} & \underset{(-)}{S_{23}} \\ 0 & \underset{(+)}{S_{32}} & \underset{(-)}{S_{33}} \end{vmatrix}$$

$$= \frac{1}{\det S}(\underset{(+)}{S_{32}}\underset{(-)}{S_{23}} - \underset{(-)}{S_{22}}\underset{(-)}{S_{33}}) > 0, \tag{62}$$

$$\frac{dY_2}{dG_{01}} = (Y^2_{G01})^* = \frac{1}{\det S}\begin{vmatrix} \underset{(-)}{S_{11}} & -1 & \underset{(+)}{S_{13}} \\ \underset{(+)}{S_{21}} & 0 & \underset{(-)}{S_{23}} \\ \underset{(-)}{S_{31}} & 0 & \underset{(-)}{S_{33}} \end{vmatrix}$$

$$= \frac{1}{\det S}(\underset{(+)}{S_{21}}\underset{(-)}{S_{33}} - \underset{(-)}{S_{23}}\underset{(-)}{S_{31}}) > 0, \tag{63}$$

$$\frac{dM_1}{dG_{01}} = (M^1_{G01})^* = \frac{1}{\det S}\begin{vmatrix} \underset{(-)}{S_{11}} & \underset{(+)}{S_{12}} & -1 \\ \underset{(+)}{S_{21}} & \underset{(-)}{S_{22}} & 0 \\ \underset{(-)}{S_{31}} & \underset{(+)}{S_{32}} & 0 \end{vmatrix}$$

$$= \frac{1}{\det S}(\underset{(-)}{S_{22}}\underset{(-)}{S_{31}} - \underset{(+)}{S_{32}}\underset{(+)}{S_{21}}) < 0. \tag{64}$$

Then, we can establish the following proposition as we have similar relationships when $i = 2$.

Proposition 4. If the parameters γ_i or μ are sufficiently large so that the absolute values of S_{11} and S_{22} as stabilizing factors of the system are larger than the absolute value of the other terms, we have the following relationships.

$$\frac{dY_1}{dG_{01}} > 0,\ \frac{dY_2}{dG_{01}} > 0,\ \frac{dM_1}{dG_{01}} < 0, \tag{65}$$

$$\frac{dY_1}{dG_{02}} > 0,\ \frac{dY_2}{dG_{02}} > 0,\ \frac{dM_1}{dG_{02}} > 0. \tag{66}$$

Proposition 4 implies that government expenditure does not only have a positive impact for national income in such countries, but also has a positive impact on other countries. Therefore, an increase in government expenditure in periphery countries can increase national income in core countries. Then, Proposition 4 also implies that a decrease in monetary demand due to increased interest rate is larger than an increase of monetary demand by an increased government expenditure in country 1.

Second, we consider the case when $d\tau_1 \neq 0$ and $dG_1 = 0$. We investigate the change in endogenous variables in (55) using a Cramer's rule.

$$\frac{dY_1}{d\tau_1} = (Y^1_{\tau_1})^* = \frac{1}{\det S}\begin{vmatrix} c_1Y_1 & \underset{(+)}{S_{12}} & \underset{(+)}{S_{13}} \\ 0 & \underset{(-)}{S_{22}} & \underset{(-)}{S_{23}} \\ 0 & \underset{(+)}{S_{32}} & \underset{(-)}{S_{33}} \end{vmatrix}$$

$$= \frac{c_1Y_1}{\det S}(\underset{(-)}{S_{22}}\underset{(-)}{S_{33}} - \underset{(+)}{S_{32}}\underset{(-)}{S_{23}}) < 0, \tag{67}$$

$$\frac{dY_2}{d\tau_1} = (Y^2_{\tau_1})^* = \frac{1}{\det S}\begin{vmatrix} \underset{(-)}{S_{11}} & c_1Y_1 & \underset{(+)}{S_{13}} \\ \underset{(+)}{S_{21}} & 0 & \underset{(-)}{S_{23}} \\ \underset{(-)}{S_{31}} & 0 & \underset{(-)}{S_{33}} \end{vmatrix}$$

$$= \frac{c_1Y_1}{\det S}(\underset{(-)}{S_{23}}\underset{(-)}{S_{31}} - \underset{(+)}{S_{21}}\underset{(-)}{S_{33}}) < 0, \tag{68}$$

$$\frac{dM_1}{d\tau_1} = (M^1_{\tau_1})^* = \frac{1}{\det S}\begin{vmatrix} \underset{(-)}{S_{11}} & \underset{(+)}{S_{12}} & c_1Y_1 \\ \underset{(+)}{S_{21}} & \underset{(-)}{S_{22}} & 0 \\ \underset{(-)}{S_{31}} & \underset{(+)}{S_{32}} & 0 \end{vmatrix}$$

$$= \frac{c_1Y_1}{\det S}(\underset{(+)}{S_{32}}\underset{(+)}{S_{21}} - \underset{(-)}{S_{22}}\underset{(-)}{S_{31}}) > 0. \tag{69}$$

Then, we can establish the following proposition as we have similar relationships when $i = 2$.

Proposition 5. If the parameters γ_i or μ are sufficiently large so that the absolute values of S_{11} and S_{22} as a stabilizing factors of the system are larger than the absolute values of the other terms, we have the following relationships.

$$\frac{dY_1}{d\tau_1} < 0,\ \frac{dY_2}{d\tau_1} < 0,\ \frac{dM_1}{d\tau_1} > 0,$$
$$\frac{dY_1}{d\tau_2} < 0,\ \frac{dY_2}{d\tau_2} < 0,\ \frac{dM_1}{d\tau_2} < 0. \tag{70}$$

Proposition 5 implies that an increased tax leads to a decrease in national income not only of the periphery countries, but also of the core countries. An increased tax rate in core countries also leads to a decrease in national income in periphery countries. Then, Proposition 5 also implies that an increase in monetary demand due to a decrease in interest rate is larger than an increase in monetary demand due to increased taxes in periphery countries.

7.3 Comparative Static Analysis of Fiscal and Monetary Policy

We develop a comparative statics analysis of fiscal and monetary policies under Assumption 1. In Eqs. (43)–(45), we investigate the impact of an endogenous variables Y_1, Y_2 and M_1 on equilibrium value if the total monetary supply of both countries $\bar{M}$ and the fiscal policy parameters τ_i and G_{0i} change by the amount $d\bar{M}$, $d\tau_i$, and dG_{0i} respectively.

$$dY_1 = (Y^1_{\bar{M}})^* d\bar{M} + (Y^1_{G01})^* dG_{01} + (Y^1_{\tau 1})^* d\tau_1, \tag{71}$$

$$dY_2 = (Y^2_{\bar{M}})^* d\bar{M} + (Y^2_{G01})^* dG_{01} + (Y^2_{\tau 1})^* d\tau_1, \tag{72}$$

$$dM_1 = (M^1_{\bar{M}})^* d\bar{M} + (M^1_{G01})^* dG_{01} + (M^1_{\tau 1})^* d\tau_1. \tag{73}$$

In this paper, we consider the following two cases.

(i) $d\bar{M} = dG_{01} > 0$, $d\tau_1 = 0$.

We assume that a supranational central bank purchases national bonds that are issued to finance government expenditure in country 1.

$$dY_1 = [\underset{(+)}{(Y^1_{\bar{M}})^*} + \underset{(+)}{(Y^1_{G01})^*}]\, \underset{(+)}{d\bar{M}} > 0, \tag{74}$$

$$dY_2 = [\underset{(+)}{(Y^2_{\bar{M}})^*} + \underset{(+)}{(Y^2_{G01})^*}]\, \underset{(+)}{d\bar{M}} > 0, \tag{75}$$

$$dM_1 = [\underset{(+)}{(M^1_{\bar{M}})^*} + \underset{(-)}{(M^1_{G01})^*}]\, \underset{(+)}{d\bar{M}}\,. \tag{76}$$

An increase in government expenditure with an issue of national bonds financed by a supranational central bank leads to an increase the national income of both country 1 and country 2 that is larger than the increase in income created by an increase of money supply without an increase in government expenditure. Thus, an increase in government expenditure with the issue of national bonds of a periphery country financed by a supranational central bank leads to an increase in national income not only of the periphery country, but also of the core country. Then, the money supply of country 1 is smaller than the money supply not accompanied an increase in government expenditure.

(ii) $d\bar{M} > 0$, $dG_{01} = 0$, $d\tau_1 > 0$.

We assume that country 1 adopts an expansionary monetary policy and increases taxation.

$$dY_1 = \underset{(+)}{(Y^1_{\bar{M}})^*}\, \underset{(+)}{d\bar{M}} + \underset{(-)}{(Y^1_{\tau 1})^*}\, \underset{(+)}{d\tau_1}, \tag{77}$$

$$dY_2 = \underset{(+)}{(Y^2_{\bar{M}})^*}\, \underset{(+)}{d\bar{M}} + \underset{(-)}{(Y^2_{\tau 1})^*}\, \underset{(+)}{d\tau_1}, \tag{78}$$

$$dM_1 = \underset{(+)}{(M^1_{\bar{M}})^*}\, \underset{(+)}{d\bar{M}} + \underset{(+)}{(M^1_{\tau 1})^*}\, \underset{(+)}{d\tau_1} > 0. \tag{79}$$

A policy mix of a monetary expansion by a supranational central bank and an increased taxation in country 1 increase national income in both country 1 and country 2 by an amount smaller than that created by a monetary expansion not accompanied by an increased tax rate. Then, the money supply of country 1 is larger than the monetary easing not accompanied an increased tax. Therefore, a policy mix of a monetary easing and an increased tax actually implemented by the ECB and the periphery country has a smaller impact than that of a monetary expansion on national income of the periphery county and the core country.

8 Conclusion

In this paper, we analyzed the effect of fiscal transfers as an insurance mechanism in a fiscal union, and the relation to a Capital Markets Union, using a Keynesian two-country model with monetary union and imperfect capital mobility. The European Commission wants to increase the size of the capital market in order to realize the benefits of a Capital Markets Union. However, the results from this paper indicate that an increase in capital mobility between countries in a Capital Markets Union can be a destabilizing force and have negative consequences for the economy of euro area. Therefore, it is important for countries in euro area to adopt counter-cyclical fiscal policies and create a fiscal union to mitigate the instability.

However, it is difficult for periphery countries to adopt expansionary fiscal policies, because periphery countries are required to obey fiscal discipline. Thus, the balance between private capital mobility and support by fiscal policy collapses. As pointed out in the theory of the optimum currency area, fiscal transfers allow for shock adjustment between countries. To enlarge size of capital market, it is necessary to construct a fiscal transfer system as an insurance mechanism by creating a fiscal union.

Furthermore, we stress that a monetary easing by a supranational central bank in a fiscal union increases the national income not only of the periphery countries but also of the core countries. In other words, a quantitative easing implemented by the ECB even in a fiscal union will have a positive impact on the economies of the euro area. In addition, we argue that an increase in government expenditure with an issue of national bonds financed by the supranational central bank enhances the positive effect on national income. Conversely, austerity fiscal policies in periphery countries decrease national income not only of periphery countries but also of core countries.

One of the limitations of this paper is that we eliminate price fluctuation. Actually, the ECB targets an increase in inflation rate and expected inflation rate by implementing an expansionary monetary policy. Therefore, a model that includes inflation rate and expected inflation rate should be constructed. This study contribute to the literature on economic stability in a fiscal union in that it defined a clear relationship between a fiscal union and a Capital Markets Union.

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