Tariffs, Vertical Oligopoly and Market Structure

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Abstract

What is the relationship between domestic competition policy and international trade policy in the presence of vertical specialization? Should the government liberalize entry in its domestic final-good market in order to enhance an effect of liberalization in input trade? To address these questions, we develop a vertical oligopoly model in which the relative thickness of upstream and downstream markets plays a key role in welfare evaluations. In our model, a Home government imposes tariffs on imported input from Foreign upstream firms, and simultaneously restricts entry of Home downstream firms. Since Home and Foreign countries are vertically interdependent in this setting, trade policy has a crucial impact not only on Foreign firms, but also on Home firms through "firm-colocation" effects. We find that, in the short-run equilibrium, the optimal tariff is higher, the thicker is the Home final-good market (relative to Foreign input market). In the long-run equilibrium, however, this relationship is overturned and the optimal tariff is higher, the thinner is the Home final-good market. This finding suggests that reduction of import tariff for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

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Very preliminary and incomplete

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1 Introduction

Recent years have witnessed much faster growth in intermediate input trade than final good. It is often argued that this rapid growth in intermediate input has triggered by the aspect that international trade allows each country to vertically specialize in only narrowly defined production by fragmenting production processes spread across the globe. Further it is also argued that this input trade growth takes place largely through foreign outsourcing in which domestic final-good producers procure some intermediate input from foreign suppliers through contracting. Vertical structures of this kind can be represented by bilateral oligopoly models with bargaining over the terms of contracts that specify the price and quantity. In reality, however, a large fraction of intermediate products are also internationally traded through markets among anonymous final-good producers and intermediate-input suppliers rather than vertical negotiation. For example, noticing that the term "outsourcing" refers to the procurement of inputs outside the firm that takes place through *both* contractual arrangements and spot markets, Spencer (2005) stresses that this distinction is important for evaluating gains from outsourcing and its relevant policy interventions for China's processing exports.

Based on our belief that bargaining is not the only means of procuring intermediate inputs in foreign outsourcing, we develop a vertical oligopoly model to capture an impact that market-based interactions between vertically related industries can have on trade policy. In our model, there exist a large number of potential entrants to a domestic downstream sector and a foreign upstream sector, where the number of entrants in each sector is either exogenous or endogenous. A domestic government imposes a tariff to foreign input transacted though markets, where the price and quantity are determined at the market-clearing levels (instead of bilateral bargaining). In this setting, we argue that there is a characterization of how optimal tariffs that maximize domestic welfare vary with the market thickness between the domestic and foreign countries. In particular, we show that, while optimal tariffs vary with the numbers of domestic downstream firms and foreign upstream firms, policy implications from the model are drastically different, depending upon whether the market structures are exogenous or endogenous.

We find that, in the short run, the optimal tariff is higher, the thicker is the domestic final-good market (relative to foreign input market). In the long run, however, this relationship is overturned and the optimal tariff is higher, the thinner is the domestic final-good market. This difference comes mainly from the fact that trade policy has a crucial impact not only on entry and exit of foreign firms, but also on domestic firms in the long run. In vertical specialization where domestic firms' output and foreign firms' input are complements, if tariff on intermediate input from foreign discourages entry of foreign firms, it also discourages entry of domestic firms. We call this a "firm-colocation" effect, which is used as an antonym to a "firm-delocation" effect in horizontal specialzation (see, e.g., Bagwell and Staiger, 2012a, b). This finding suggests that reduction of import tariff for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

2 Model

Consider a setting with two countries, Home and Foreign, specializing respectively in a final good and an intermediate input. Foreign has n identical upstream firms, $F_1, F_2, ..., F_n$. Home has m identical downstream firms, $H_1, H_2, ..., H_m$. In the upstream sector in Foreign, a homogeneous intermediate input is produced with constant marginal cost c and shipped to Home with a specific tariff rate t. In the downstream sector in Home, the imported intermediate input is transformed into a homogeneous final good with constant marginal cost c_d, which is normalized to zero for simplicity. In addition to these production costs, upon entry, Home and Foreign firms incur fixed entry costs K_H and K_F respectively.

There is a unit mass of identical consumers with a quasi-linear utility function, $U(Q) + y$, where Q is a imperfectly competitive final good produced by using an intermediate input and y is a perfectly competitive numeraire good.[1] Assuming income to be high enough, maximizing $U(Q)+y$ subject to the budget constraint gives demand for the homogeneous product: $Q = Q(P)$. Assume the preferences are such that (i) $Q(P)$ is twice continuously differentiable and $Q'(P) < 0$ for all $P \in (0, \bar{P})$ where $\bar{P} \equiv \lim_{Q \to 0} P^{-1}(Q)$ and (ii) $Q(P) = 0$ for $P \geq \bar{P}$. These assumptions guarantee the existence of the Cournot equilibrium. We will often work with inverse demand functions. These assumptions regarding $Q(P)$ imply that the inverse demand function $P = P(Q)$ is twice continuously differentiable and $P'(Q) < 0$ for all $Q \geq 0$. For a sharper characterization, we assume that the final goods are consumed only in Home and that the Foreign government does not undertake trade policy, but none of the key results relies on these assumptions.

We consider the three-stage game. In the first-stage, the Home government sets a specific tariff rate, t, to maximize Home welfare which consists of consumer surplus, aggregate Home profits and tariff revenues. In the second stage, upon paying the fixed entry cost K_F, Foreign firms enter the market and engage in a Cournot competition in the upstream sector where profit-maximizing upstream firms commit to choose the quantity of the intermediate input taking rival firms' input as given. In the third stage, upon paying the fixed entry cost K_H, Home firms enter the market and engage in a Cournot competition in the downstream sector where profit-maximizing downstream firms commit to choose the quantity of the final good taking rival firms' output and the input price (denoted by r) as given. The input price r is determined at the market clearing level which equals the total amount of the intermediate input demanded by downstream firms to the total amount of the intermediate input supplied by the upstream firms.

In order to illustrate important policy implications and empirically testable predictions, we conduct both the "short-run" analysis and the "long-run" analysis in a unified framework. In the short-run analysis in Section 3, we bypass entry considerations in both sectors of production and assume that the numbers of Home and Foreign firms are fixed. Thus, in this section, tariff

[1] As is well-known, if this numeraire is freely tradable across Home and Foreign, wage rates between these tow countries are equalized, allowing us to interpret the model in general-equilibrium terms. Since labor-market-clearing conditions are not explicitly analyzed in this setting, we implicitly assume that common wage rates are unity whereby all production costs and entry costs are measured by this numeraire.

has no effect on the market structure. In Section 4, by contrast, we assume that after observing tariff rates, firms enter the market. Thus, in this section, the market structure is endogenous in that tariff changes the numbers of Home and Foreign firms as well as the outputs of these firms.

3 Exogenous Market Structure

This section considers an environment where the market structure is given. The entry costs K_H and K_F have been sunk and entry of Home and Foreign firms has taken place. Thus, we treat the numbers of these firm m, n as fixed and invariant to the tariff rate. In what follows, we derive the Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model described in the previous section. Formal proofs for all propositions and lemmas are relegated to the appendix.

3.1 Cournot Competition

We first analyze the third-stage Cournot competition among Home firms in the final-good market. Each Home firm H_i chooses q_i to maximize $P\left(\left(q_i + \sum_{j\neq i}^{m} q_j\right) - r\right) q_i$ taking other downstream firms' outputs and input price $r(< \bar{P})$ as given. If $q_i > 0$ for all $i = 1, 2, ..., m$, the first-order conditions are

$$P\left(q_i + \sum_{j\neq i}^{m} q_j\right) - r + P'\left(q_i + \sum_{j\neq i}^{m} q_j\right) q_i = 0.$$

The assumption below ensures that the solution to the maximization problem is unique.

Assumption 1 The demand function $Q(P)$ is logconcave.

The equivalent assumption in terms of inverse demand function is:

Assumption 1' $P'(Q) + QP''(Q) \leq 0$ for all $Q \geq 0$.

Assumption 1 holds if and only if marginal revenue is steeper than demand. In the trade literature, this assumption is first introduced in Brander and Spencer (1984a, b) who show that when the Home country imports from a Foreign monopolist with constant marginal cost, a small tariff improves welfare if and only if Assumption 1' holds.

In our framework, in addition to guaranteeing uniqueness, Assumption 1' ensures that the optimal tariff is non-negative at least for some $m > 1$ and $n > 1$. A convenient way to state Assumption 1' is in terms of elasticity of slope which is defined as $\epsilon(Q) \equiv \frac{QP''(Q)}{P'(Q)}$. Observe that $\epsilon(Q) \geq -1 \Leftrightarrow P'(Q) + QP''(Q) \leq 0$. This condition is sufficient to prove the main results. For analytical simplicity, we focus on a class of demand functions which not only satisfy Assumption 1 but also satisfy the following:

Assumption 2 $\epsilon(Q) = \frac{QP''(Q)}{P'(Q)} = \epsilon$.

Note, if ϵ is constant for all $Q(\geq 0)$, ϵ is greater than -1 and Assumption 1' or Assumption 1 is satisfied as well. Although this assumption is admittedly restrictive, any well-known inverse demand function satisfies Assumption 2: linear, constant elasticity, and semi-log among others.[2]

Now back to the Cournot competition in the downstream sector. If $r \in (0, \bar{P})$, Assumption 1 or 1' guarantees that there exists a unique symmetric equilibrium $\hat{q}_1 = ... = \hat{q}_m = \hat{q}(> 0)$ such that

$$\hat{q} = -\frac{P(\hat{Q}) - r}{P'(\hat{Q})},$$

where $\hat{Q} = m\hat{q}$ is uniquely solves the following equation:

$$mP(\hat{Q}) + \hat{Q}P'(\hat{Q}) = mr. \tag{3.1}$$

Let $\pi_H(q, \hat{q}) \equiv [P(q + (m-1)\hat{q}) - r]q$ denote the post-entry profit of a downstream firm that chooses q as its quantity given all other $m-1$ firms choose $\hat{q}$. Suppose $\pi_H(q, \hat{q})$ is pseudoconcave in q at $q = \hat{q}$. If $r \in (0, \bar{P})$, we have that $\hat{q}_1 = ... = \hat{q}_m = \hat{q}(> 0)$ constitutes the Stage 3 equilibrium. On the other hand, if $r \in [\bar{P}, \infty)$, each downstream firm i chooses $q_i = 0$ in the Stage 3 equilibrium (see Ghosh and Morita (2007) for details).

Let X denote the aggregate input demanded at any given input price $r \in (0, \bar{P})$. Since one unit of final good requires one unit of intermediate input, we have that $X = \hat{Q} = m\hat{q}$. Further since the input price is determined at the market clearing level and the aggregate amount of final good produced at any given $r \in (0, \bar{P})$ is $\hat{Q}$, it follows from (3.1) that the inverse demand function for intermediate good X faced by upstream firms is given by

$$r = P(Q) + \frac{QP'(Q)}{m} \equiv g(X).^{3} \tag{3.2}$$

From $P'(Q) + QP''(Q) \leq 0$ (by Assumption 1'), we have that

$$g'(X) = \frac{(m+1)P'(Q) + QP''(Q)}{m} = \frac{P'(Q)(m+1+\epsilon)}{m} < 0. \tag{3.3}$$

Moreover, from $\epsilon(Q) = \frac{QP''(Q)}{P'(Q)} = \epsilon$ for all $Q \geq 0$ (by Assumption 2), we also have that

$$\frac{Xg''(X)}{g'(X)} = \frac{\frac{QP''(Q)(m+1+\epsilon)}{m}}{\frac{P'(Q)(m+1+\epsilon)}{m}} = \epsilon.$$

[2] Ara and Ghosh (2016) analyze trade policy in matching markets by using more general demand functions:

$$\frac{\epsilon'(Q)Q}{\epsilon(Q)} \leq 1 \iff \frac{P''(Q)Q}{P'(Q)} \geq \frac{P'''(Q)Q}{P''(Q)},$$

which implies the curvature of inverse demand is greater than the curvature of slope of inverse demand for all $Q \geq 0$.

[3] From (3.2), it follows that r depends not only on X but also on m and thus it is more precise to define $r \equiv g(X, m)$. While we apply this short-hand definition $r \equiv g(X)$ for the short-run analysis (since we mainly focus on comparative statics with respect to n), this distinction becomes important in the long-run analysis.

Thus, the inverse demand function for intermediate good is downward-sloping and the elasticity of slope of input demand is the same as that of final-good demand. Note that Assumption 1 implies $\epsilon \geq -1$ which in turn implies for all $X \geq 0$ that

$$g'(X) + Xg''(X) \leq 0. \tag{3.4}$$

Now consider the second-stage Cournot competition among Foreign firms in the intermediate-good market. The inverse demand function faced by upstream firms at Stage 2 is given by (3.2). Each Foreign firm F_i chooses x_i to maximize $\left[g\left(x_i + \sum_{j\neq i}^{n} x_j\right) - c - t\right] x_i$ taking other upstream firms' inputs as given. If $x_i > 0$ for all $i = 1, 2, ..., n$, the first-order conditions are

$$g\left(x_i + \sum_{j\neq i}^{n} x_j\right) - c - t + g'\left(x_i + \sum_{j\neq i}^{n} x_j\right)x_i = 0.$$

Given that $\lim_{X\to 0} g(X) = \bar{P}$ from (3.2), condition (3.4) (which is analogous to Assumption 1') guarantees that there exists a unique equilibrium $\hat{x}_1 = ... = \hat{x}_n \equiv \hat{x}(> 0)$ such that

$$\hat{x} = -\frac{g(\hat{X}) - c - t}{g'(\hat{X})},$$

where $\hat{X} = n\hat{x}$ uniquely solves the following equation:

$$ng(\hat{X}) + g'(\hat{X})\hat{X} = n(c + t). \tag{3.5}$$

Let $\pi_F(x, \hat{x}) \equiv [g(x + (n-1)\hat{x}) - c - t]x$ denote the post-entry profit of an upstream firm that chooses x as its quantity given all other $n-1$ firms choose $\hat{x}$. Since $\pi_F(x, \hat{x})$ is strictly concave in x for all $x > 0$ (by virtue of (3.4)) and $r \in (0, \bar{P})$, we have that $\hat{x}_1 = ... = \hat{x}_n = \hat{x}(> 0)$ constitutes the Stage 2 equilibrium.

To summarize, in the Cournot competition with given m, n and t, we have an output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$ and a price vector $(\hat{P}, \hat{r})$ where

- $\hat{Q}$ solves (3.1);
- $\hat{X}$ solves (3.5);
- $\hat{Q} = \hat{X}$;
- $\hat{q} = \frac{\hat{Q}}{m}$, $\hat{x} = \frac{\hat{X}}{n}$;
- $\hat{P} \equiv P(\hat{Q})$, $\hat{r} \equiv g(\hat{X})$.

The following lemma records some comparative statics results with respect to n and t.[4]

[4] In this section, we omit comparative statics with respect to m just for simplicity. In the Appendix, we also show these comparative statics.

Lemma 3.1

(i) *For a given tariff rate* t*, the aggregate output* $\hat{Q}$ *and aggregate input* $\hat{X}$ *are increasing in* n*; while the final-good price* $\hat{P}$ *and input price* $\hat{r}$ *are decreasing in* n*; i.e.,* $\partial \hat{Q}/\partial n = \partial \hat{X}/\partial n > 0$*,* $\partial \hat{P}/\partial n < 0$*,* $\partial \hat{r}/\partial n < 0$*.*[5]

(ii) *For a given number of firms* m, n*, the aggregate output* $\hat{Q}$ *and aggregate input* $\hat{X}$ *are decreasing in* t*; while the final-good price* $\hat{P}$ *and input price* $\hat{r}$ *are increasing in* t*; i.e.,* $\partial \hat{Q}/\partial t = \partial \hat{X}/\partial t < 0$*,* $\partial \hat{P}/\partial t > 0$*,* $\partial \hat{r}/\partial t > 0$*.*

(iii) *Let* $r^* \equiv \hat{r} - t$ *denote the price received by a Foreign firm in equilibrium (for each unit of the intermediate input). Then,*

$$\frac{\mathrm{d}r^*}{\mathrm{d}t} \lesseqgtr 0 \quad \Leftrightarrow \quad \frac{\mathrm{d}\hat{r}}{\mathrm{d}t} \lesseqgtr 1 \quad \Leftrightarrow \quad 1 + \epsilon \gtreqless 0.$$

Not surprisingly, $\hat{r}$ increases as t increases. However, $\frac{\mathrm{d}\hat{r}}{\mathrm{d}t} - 1 \leq 0$ or equivalently $\frac{\mathrm{d}r^*}{\mathrm{d}t} \leq 0$ as long as the demand is logconcave. For all such demand functions, the pass-through of tariff to an intermediate-input price faced by Home producers is less than complete. Foreign firms absorb part of the tariff increase which acts like a terms-of-trade gain for Home. While r^* is an input price internal to the firms, a reduction in r^* hurts Foreign firms and benefits Home firms. Hence, we refer to a decrease in r^* as an improvement in terms-of-trade in the paper, though we are aware that r^* is more like firms' terms-of-trade (rather than countries' terms-of-trade). We introduce the concept of an input price since it can be interpreted in a similar fashion to the terms-of-trade. The terms-of-trade improvement creates a rationale for Home to set a positive tariff.

Note that, like the vertical oligopoly models developed by Ishikawa and Lee (1997) and Ishikawa and Spencer (1999), there is a "double marginalization" effect at work in our model: imperfect competition in both the final-good market and intermediate-good market simultaneously creates a wedge between the prices of final good and intermediate input and their marginal cost. The inefficiency associated with this double marginalization effect in vertical oligopolies also influences the optimal tariff set by Home.

[5]Note that (3.1) and (3.5) alternatively define $\hat{q}$ and $\hat{x}$ respectively in a unique symmetric equilibrium:

$$P(m\hat{q}) + \hat{q}P'(m\hat{q}) = r,$$
$$g(n\hat{x}) + \hat{x}g'(n\hat{x}) = c + t,$$

where $m\hat{q} = \hat{Q}$ and $n\hat{x} = \hat{X}$. Further, by invoking the Implicit Function Theorem to these equations, we have that $\hat{q}$ and $\hat{x}$ are a continuously differentiable function of n:

$$\frac{\partial \hat{q}}{\partial n} > 0, \quad \frac{\partial \hat{x}}{\partial n} < 0,$$

which imply that an increase in n leads to the "business-creating" effect in the downstream sector whereas it also leads to the "business-stealing" effect arises in upstream sector.

3.2 Tariffs

In the first stage, the Home government chooses a tariff rate t to maximize Home welfare (W_H), taking the output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$ and the price vector $(\hat{P}, \hat{r})$ as given. In the SPNE of the Stage 1 subgame, W_H is given by

$$W_H \equiv \underbrace{\left[\int_0^{\hat{Q}} P(y)\mathrm{d}y - P(\hat{Q})\hat{Q}\right]}_{\text{Consumer surplus } (CS)} + \underbrace{\left[P(\hat{Q}) - \hat{r}\right]\hat{Q}}_{\text{Home profits } (\Pi_H)} + \underbrace{t\hat{X}}_{\text{Tariff revenue } (TR)}.^{6}$$

where $\Pi_H \equiv m\pi_H = (\hat{P} - \hat{r})\hat{Q}$. Using $r^* = \hat{r} - t$ and simplifying the above expression gives

$$W_H \equiv \int_0^{\hat{Q}} P(y)\mathrm{d}y - r^*\hat{X}.$$

Differentiating W_H with respect to t and using $\frac{\partial \hat{Q}}{\partial t} = \frac{\partial \hat{X}}{\partial t}$, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - r^*)\frac{\partial \hat{Q}}{\partial t} - \frac{\partial r^*}{\partial t}\hat{X},$$

The first term captures the welfare loss due to the tariff-induced output reduction ($\frac{\partial \hat{Q}}{\partial t} < 0$). Home consumers value the good at $P(\hat{Q})$ while effectively it costs $r^*(< P(\hat{Q}))$ to produce (from Home's perspective). This price-cost margin $P(\hat{Q}) - t^*$ multiplied by the amount of output lost $\frac{\partial \hat{Q}}{\partial t}$ is the magnitude of welfare loss. The second term captures the welfare gains arising from the terms-of-trade improvement ($\frac{\partial r^*}{\partial t} < 0$). The optimal tariff rate strikes a balance between the two competing effects – welfare gains from the terms-of-trade improvement and welfare losses from the reduction in output. As we show below, the number of firms in each sector of production m, n plays an important role in delineating the relative importance of the two effects, which in turn helps to determine the sign of the optimal tariff.

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ and solving for t gives the expression for the optimal tariff which is presented later in Proposition 3.1. Here we first focus on the sign of the optimal tariff. Using $\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1$, we can express $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ as follows:

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} + t\frac{\partial \hat{X}}{\partial t}. \quad (3.6)$$

Using (3.6) and noting that $\frac{\partial \hat{X}}{\partial t} < 0$, the optimal tariff is strictly positive (negative) if and only if

$$(P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} > (<)0. \quad (3.7)$$

[6]For the class of inverse demand functions that satisfy Assumptions 1 and 2, W_H is strictly concave in t so that the second-order condition is satisfied, i.e., $\frac{\partial^2 W_H}{\partial t^2} < 0$.

Equation (3.7) indicates that the number of firms plays a key role in determining the sign of the optimal tariff. To see this, suppose that for a given m, the number of Foreign firms n is arbitrarily large so that the intermediate-input market becomes perfectly competitive. Then, the input price equals its marginal cost ($\hat{r} = c + t$) and, as a result, $\left(1 - \frac{\partial \hat{r}}{\partial t}\right) \hat{X} = -\frac{\partial r^*}{\partial t} \hat{X} = 0$, i.e., the terms-of-trade motive vanishes. Only the harmful effect of the tariff – output reduction – remains. An import subsidy raises Home welfare by increasing output and indeed the optimal tariff is negative. More generally, when n is arbitrarily large, Home captures all profits in Cournot competition of the downstream market. The situation is like a domestic, single-stage, Cournot oligopoly with m firms. A positive subsidy increases welfare in an oligopoly setup by narrowing the wedge between price and marginal cost, which explains why an import subsidy is optimal.

For the other extreme case, suppose that for a given n, the number of Home firms m is arbitrarily large so that the final-good market becomes perfectly competitive. Then, the final-good price equals its marginal cost ($P(\hat{Q}) = \hat{r}$) and, as a result, $(P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} = 0$, i.e., the welfare loss due to the tariff-induced output reduction vanishes. This is equivalent for Home to importing the final good from Foreign and its welfare is composed of the consumer surplus and tariff revenues. In such a case, the sign of the optimal tariff is determined exclusively by the terms-of-trade motive, or equivalently by the sign of $1 - \frac{\partial \hat{r}}{\partial t} = -\frac{\mathrm{d}r^*}{\mathrm{d}t}$. As the pass-through from tariff to domestic prices is incomplete for all logconcave demand functions, $1 - \frac{\partial \hat{r}}{\partial t} > 0$ holds, which implies that the optimal tariff is strictly positive.

The above intuition suggests that Home's optimal tariff is positive when the number of Foreign firms (n) is relatively smaller than the number of Home firms (m), and it is negative when n is relatively larger than m. This comes out more cleanly in terms of the price-cost margin ratio $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$. Note that, in the presence of the double marginalization effect, when this ratio is small (large), the final-good market is more (less) competitive relative to the intermediate-input market. Using (3.2) and (3.5), this ratio can be rewritten as

$$\frac{\hat{P} - \hat{r}}{\hat{r} - c - t} = \frac{-\frac{\hat{Q}P'(\hat{Q})}{m}}{-\frac{\hat{X}g'(\hat{X})}{n}} = \frac{n}{m + 1 + \epsilon}, \tag{3.8}$$

which is increasing in n. Further, invoking the standard continuity argument, there is a range of values such that the optimal tariff is strictly decreasing in n. Analyzing (3.6) further gives a more precise characterization.

Proposition 3.1

Let $t(n)$ denote the optimal tariff. At $t = t(n)$ the following holds:

$$t = -\hat{Q}P'(\hat{Q}) \left(\frac{(1 + \epsilon)(m + 1 + \epsilon) - n}{mn} \right), \tag{3.9}$$

where $\hat{Q}$ is the aggregate output evaluated at $t = t(n)$. Furthermore,

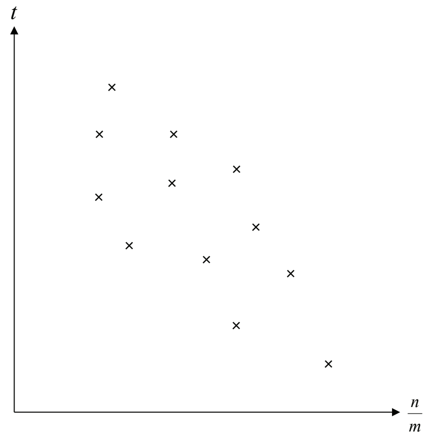

FIGURE 3.1 – Optimal tariff in short run

(i) *There exists* n^* *such that*

$$t(n) \gtreqless 0 \quad \Leftrightarrow \quad n \lesseqgtr n^* \equiv (1+\epsilon)(m+1+\epsilon).$$

(ii) $t(n)$ *is monotonically decreasing in* n*.*

As an illustrative example, consider the following class of inverse demand functions: $P(Q) = a - Q^b, b > 0$. Observe that $b = 1$ for linear demand and $b > (<)1$ for strictly concave (convex) demand. The elasticity of slope is constant and denoted by $\epsilon = b - 1$. Applying (3.8) yields

$$t = \frac{(a-c)b}{mn + b(b+1)(m+b)} \left(b(m+b) - n\right).$$

Note the property of the optimal tariff in Proposition 3.1 holds for this specific demand function. In addition, the optimal tariff rate is higher when market size is greater ($\frac{\partial t}{\partial a} > 0$) and demand is more concave ($\frac{\partial t}{\partial b} > 0$).

While we focus on how the number of Foreign firms n affect the optimal tariff t in Proposition 3.1, it is straightforward to show that the similar result holds for the number of Home firms m. The above intuition indeed tells us that the optimal tariff is increasing in m. This in turn helps consider how the relative number of firms $\frac{n}{m}$ – which is hereafter referred to as "relative market thickness" – affects the optimal tariff. Since the optimal tariff is increasing (decreasing) in m (n), our model predicts that the optimal tariff should be decreasing in $\frac{n}{m}$. Thus, if $\frac{n}{m}$ varies across industries, there would exist a negative relationship between $\frac{n}{m}$ and t in the short run.[7] Figure 3.1 illustrates our prediction when the optimal tariff is positive.

[7]Empirical evidence on market-thickness effects has been documented in law and economics. For instance, Pirrong (1993) provide evidence that thicker markets tend to lower transactions costs.

3.3 Profits

Home and Foreign aggregate profits respectively are given by

$$\Pi_H = m\pi_H = [P(\hat{Q}) - \hat{r}]\hat{Q}, \quad \Pi_F = n\pi_F = (r(\hat{X}) - c - t)\hat{X},$$

where $\pi_H = (P(\hat{Q}) - \hat{r})\hat{q}$ and $\pi_F = (\hat{r} - c - t)\hat{x}$ denote respectively the post-entry profit of each downstream and upstream firm in the SPNE of the Stage 1 subgame. Note that π_H and π_F are continuous in n and strictly increasing and decreasing in n respectively. Here we examine the impact of an increase in n on Π_H and Π_F. Differentiating Π_H with respect to n yields

$$\frac{\mathrm{d}\Pi_H}{\mathrm{d}n} = -(2+\epsilon)\hat{q}P'(\hat{Q})\frac{\partial \hat{Q}}{\partial n} > 0.$$

Not surprisingly, an increase in n has a positive effect on Π_H by lowering the input price r. On the other hand, differentiating Π_F with respect to n yields

$$\frac{\mathrm{d}\Pi_F}{\mathrm{d}n} = \underbrace{(n-1)\hat{x}g'(\hat{X})\frac{\partial \hat{X}}{\partial n}}_{\text{competition effect}} \quad \underbrace{-\frac{\partial t}{\partial n}\hat{X}}_{\text{tariff-reduction effect}} \quad .$$

An increase in n has two opposing effects on Π_F. First, an increase in n reduces the input price r and lowers Foreign profits. We call this the *competition effect*, which is captured by the first term in the above expression. Note that this effect exists even when the tariff is exogenously set. Second, an increase in n lowers t and leads to higher Foreign profits. We call this indirect effect the *tariff-reduction effect*, which is captured by the second term in the above expression. Surprisingly, for arbitrarily large m, the latter effect dominates the former and Π_F rises as n increases if the number of Foreign firms is small or the inverse demand is sufficiently concave.

Proposition 3.2

An increase in the number of Foreign firms might lead to higher Foreign profits. For arbitrarily large m, $\frac{\mathrm{d}\Pi_F}{\mathrm{d}n}\big|_{m=\infty} > 0$ if

$$n < \frac{1+\sqrt{1+4(1+\epsilon)(2+\epsilon)}}{2}.$$

Proposition 3.2 suggests that an indirect increase in Foreign profits due to a lower tariff on intermediate inputs (induced by larger n) might outweigh a direct decrease in Foreign profits due to more competition in the upstream market. This situation is more likely when the number of Foreign firms n is small or the curvature of the inverse demand ϵ is big. To see this clearly, consider $P(Q) = a - Q^b$ for which $\epsilon = b - 1$. For linear demand ($b = 1$), $\lim_{m\to\infty} \frac{\mathrm{d}\Pi_F}{\mathrm{d}n} \gtreqless 0$ if and only if $n \lesseqgtr 2$. This implies that for sufficiently large m, Foreign profits increase as the number of Foreign firms increases from one to two. As demand functions become more concave, this counter-intuitive outcome becomes more likely.

4 Exogenous Market Structure

In Section 3, we have assumed that the numbers of Home and Foreign firms are fixed. Since m and n are fixed, these numbers do not vary with tariff rates. Now we consider an environment where m and n are endogenously determined and tariffs are set prior to entry decisions. Here, in addition to the the direct effect on quantities and prices, tariffs also indirectly affect quantities and prices by influencing the market structure. In this setting, we address the following questions: What is the relationship between domestic competition policy and international trade policy in the presence of vertical specialization? Should the Home government liberalize entry in its final-good market in order to enhance an effect of liberalization in input trade?

In the context of single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985) and more recently Etro (2011) and Bagwell and Staiger (2012a, b) all have shown that the endogenous market structure can drastically alter the optimal trade policy obtained from the exogenous market structure. Like these preceding papers, we also find that free entry can affect the optimal tariff. Recall in the short-run equilibrium that the optimal tariff is higher, the thicker is the final-good market in Home (relative to the input market in Foreign). In the long-run equilibrium, by contrast, we show that this relationship is overturned and the optimal tariff is higher, the thinner is the Home final-good market. This finding suggests that reduction of import tariff for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

The timing of events is as outlined in the last paragraph of Section 2. In Stage 1, the Home government chooses a tariff rate t, following which entry occurs. In Stage 2, upon paying a fixed entry cost K_F, Foreign firms enter in the upstream sector and engage in Cournot competition taking other upstream firms' inputs as given. In Stage 3, upon paying a fixed entry cost K_H, Home firms enter in the downstream sector and engage in Cournot competition taking other downstream firms' outputs and input price r as given. As before, we derive the Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model and focus on a class of demand functions which satisfy Assumptions 1 and 2.

4.1 Cournot Competition

Let us start with analyzing Stage 3. The Cournot competition works exactly the same way as before and the unique equilibrium in this stage is characterized by $\hat{q}_1 = \hat{q}_2 = ... = \hat{q}_m \equiv \hat{q}$ such that

$$\hat{q} = -\frac{P(\hat{Q}) - r}{P'(\hat{Q})},$$

where $\hat{Q}$ satisfies the following for any given m:

$$mP(\hat{Q}) + \hat{Q}P'(\hat{Q}) = mr. \tag{4.1}$$

In addition to (4.1), the number of Home firms m is endogenously determined as there is free entry of firms. Recall from section 2 that the entry cost of a Home firm is K_H. In the long run where entry is unrestricted, entry occurs until the post-entry profit of Home firms equals the entry cost. Let $\pi_H(m) = (P(mq) - r)q$ denote the post-entry profit of Home firms in the SPNE of the Stage 3. Then the free entry condition in the downstream sector is given by $\pi_H(\hat{m}) = K_H$:

$$[P(\hat{m}q) - r]q = K_H.$$

Aggregating this condition for all $\hat{m}$ Home firms, $\hat{m}$ satisfies the following for any given Q:

$$[P(Q) - r]Q = \hat{m}K_H. \tag{4.2}$$

We assume that $K_H \leq \pi_H(1) \equiv \bar{K}_H$, which guarantees that at least one Home firm enters in the equilibrium.

Assumption 3 $K_H \leq \bar{K}_H$.

Since π_H is continuous in m and strictly decreasing in m for all $m > 1$, Assumption 3 also ensures that $\hat{m}$ uniquely exists in the SPNE of the Stage 3 subgame.

In Stage 2, the Cournot competition works exactly the same way as before. Noting that (3.2) holds in both the short-run and long-run equilibria, the inverse demand function for intermediate good X faced by upstream firms is given by

$$r = P(Q) + \frac{QP'(Q)}{m} \equiv g(X, m),$$

which satisfies

$$\begin{aligned} g_x(X, m) &\equiv \frac{\partial g(X, m)}{\partial X} = \frac{(m + 1 + \epsilon)P'(Q)}{m} < 0, \\ g_m(X, m) &\equiv \frac{\partial g(X, m)}{\partial m} = -\frac{QP'(Q)}{m^2} > 0, \\ g_{xm}(X, m) &\equiv \frac{\partial^2 g(X, m)}{\partial X \partial m} = -\frac{(1 + \epsilon)P'(Q)}{m^2} > 0. \end{aligned}$$

Using the expression of input price $r = g(X, m)$,[8] the unique equilibrium in this stage is characterized by $\hat{x}_1 = \hat{x}_2 = ... = \hat{x}_n \equiv \hat{x}$ such that

$$\hat{x} = -\frac{g(\hat{X}, m) - c - t}{g_x(\hat{X}, m)},$$

[8] We define $r = g(X)$ in the short run (see (3.2)) as the main focus is on comparative statics with respect to n. Here we explicitly define r as a function of m as well as X since m is endogenous in the long run.

where $\hat{X}$ satisfies the following for any given n:

$$ng(\hat{X}, m) + g_x(\hat{X}, m)\hat{X} = n(c + t). \tag{4.3}$$

In addition to (4.3), the number of Foreign firms n is also endogenously determined by equaling entry occurs until the post-entry profit of Foreign firms to the entry cost. Let $\pi_F(n) = (g(nx) - c - t)x$ denote the post-entry profit of Foreign firms in the SPNE of the Stage 2. Then the free entry condition in the upstream sector is given by $\pi_F(\hat{n}) = K_F$:

$$[g(\hat{n}x, m) - c - t]x = K_F.$$

Aggregating this condition for all $\hat{n}$ Foreign firms, $\hat{n}$ satisfies the following for any given X:

$$[g(X, m) - c - t]X = \hat{n}K_F. \tag{4.4}$$

We assume that $K_F \leq \pi_F(1) \equiv \bar{K}_F$, which guarantees that at least one Foreign firm enters in the equilibrium. By applying the similar claim, this also ensures that $\hat{n}$ uniquely exists in the SPNE of the Stage 2 subgame.[9]

Assumption 4 $K_F \leq \bar{K}_F$.

To summarize, in the Cournot competition with given K_H, K_F and t, we have an output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$, a price vector $(\hat{P}, \hat{r})$ and a number vector $(\hat{m}, \hat{n})$ where

- $\hat{Q}$ solves (4.1);
- $\hat{X}$ solves (4.3);
- $\hat{m}$ solves (4.2);
- $\hat{n}$ solves (4.4);
- $\hat{Q} = \hat{X}$;
- $\hat{q} = \frac{\hat{Q}}{\hat{m}}, \hat{x} = \frac{\hat{X}}{\hat{n}}$;
- $\hat{r} \equiv g(\hat{X}, \hat{m}),\ \hat{P} \equiv P(\hat{Q})$.

Note that (4.1) and (4.3) are the *market clearing* (*MC*) conditions that hold even in the short run, whereas (4.2) and (4.4) are the *free entry* (*FE*) conditions that hold only in the long run. These two conditions jointly pin down the number of firms as well as the output of these firms in the long-run equilibrium.

[9]Following the previous section, we can show that $\hat{q}_1 = ... = \hat{q}_m = \hat{q}(> 0)$ constitutes the Stage 3 equilibrium whereas $\hat{x}_1 = ... = \hat{x}_n = \hat{x}(> 0)$ constitutes the Stage 2 equilibrium.

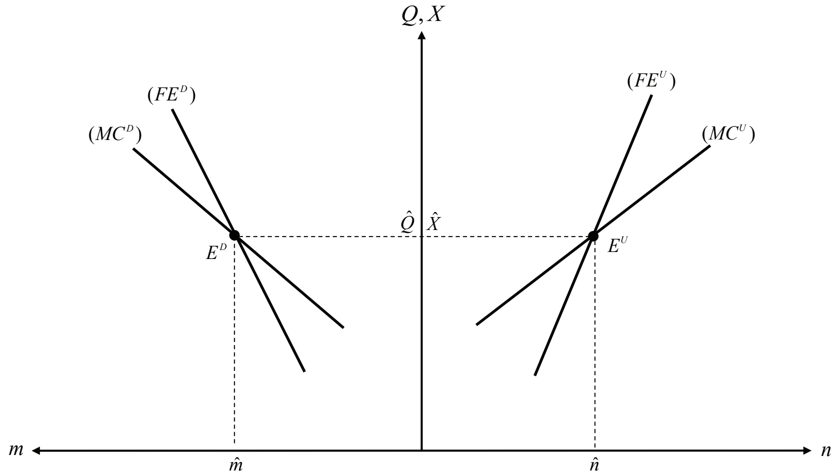

FIGURE 4.1 – Equilibrium outcomes

Figure 4.1 illustrates the equilibrium outcomes which can be solved from the ***MC*** and ***FE*** conditions. An equilibrium in the SPNE of the Stage 3 subgame is a vector $(\hat{Q}, \hat{m})$, which solves (4.1) and (4.2) in the downstream sector in Home. The second quadrant of Figure 4.1 depicts the relationship between Q and m, where (4.1) and (4.2) are given by MC^D and FE^D respectively. The fact that FE^D is steeper than MC^D follows from noting that

$$\left.\frac{\mathrm{d}Q}{\mathrm{d}m}\right|_{FE^D} = \frac{2q}{2+\epsilon} > \left.\frac{\mathrm{d}Q}{\mathrm{d}m}\right|_{MC^D} = \frac{q}{m+1+\epsilon}.$$

Point E^D, the intersection of MC^D and FE^D, uniquely determines the equilibrium vector $(\hat{Q}, \hat{m})$. From (4.2) and $\hat{Q} = \hat{m}\hat{q}$, it follows that $\hat{m}$ and $\hat{q}$ are given by

$$\hat{m} = \sqrt{-\frac{P'(\hat{Q})\hat{Q}^2}{K_H}}, \qquad \hat{q} = \sqrt{-\frac{K_H}{P'(\hat{Q})}}.$$

Similarly, an equilibrium in the SPNE of the Stage 2 subgame is a vector $(\hat{X}, \hat{n})$, which solves (4.3) and (4.4) in the upstream sector in Foreign. The first quadrant of Figure 4.1 depicts the relationship between X and n, where (4.3) and (4.4) are given by MC^U and FE^U respectively. The fact that FE^U is steeper than MC^U follows from noting that

$$\left.\frac{\mathrm{d}X}{\mathrm{d}n}\right|_{FE^U} = \frac{2x}{2+\epsilon} > \left.\frac{\mathrm{d}X}{\mathrm{d}n}\right|_{MC^U} = \frac{x}{n+1+\epsilon}.$$

Point E^U, the intersection of MC^U and FE^U, uniquely determines the equilibrium vector $(\hat{X}, \hat{n})$, where

$$\hat{n} = \sqrt{-\frac{g_x(\hat{X}, \hat{m})\hat{X}^2}{K_F}}, \qquad \hat{x} = \sqrt{-\frac{K_F}{g_x(\hat{X}, \hat{m})}}.$$

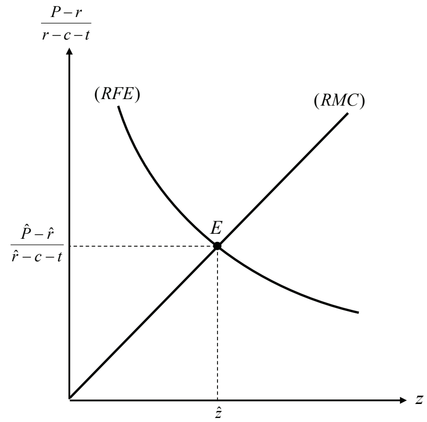

FIGURE 4.2 – Market thickness and price-cost margin

It is useful to work with the *relative market clearing* (*RMC*) condition and *relative free entry* (*RFE*) condition. Dividing (4.1) by (4.3) yields the *RMC* condition:

$$\frac{P(Q)-r}{g(X,m)-c-t} = \left(\frac{P'(Q)}{g_x(X,m)}\right) z, \tag{4.5}$$

which holds even in the short run.[10] Further, dividing (4.2) by (4.4) yields the *RFE* condition:

$$\frac{P(Q)-r}{g(X,m)-c-t} = \frac{k}{z}, \tag{4.6}$$

where $z \equiv \frac{n}{m}$ is the relative thickness of markets and $k \equiv \frac{K_H}{K_F}$ is the relative fixed cost of entry. This latter condition (4.6) holds only in the long run.

Figure 4.2 illustrates the equilibrium outcome which can be solved from the *RMC* and *RFE* conditions. Noting $r = g(X,m)$, the figure depicts the relationship between z and $\frac{P-r}{r-c-t}$, where (4.5) and (4.6) are given by *RMC* and *RFE* respectively. The fact that *RMC* is upward-sloping and *RFE* is downward-sloping directly follows from (4.5) and (4.6). Point E, the intersection of *RMC* and *RFE*, uniquely determines the equilibrium vector $(\hat{z}, \frac{\hat{P}-\hat{r}}{\hat{r}-c-t})$. Noting that the system of equations (4.5) and (4.6) can be solved for this vector, we get

$$\hat{z} = \sqrt{k\frac{g_x(\hat{X},\hat{m})}{P'(\hat{Q})}},$$

$$\frac{\hat{P}-\hat{r}}{\hat{r}-c-t} = \sqrt{k\frac{P'(\hat{Q})}{g_x(\hat{X},\hat{m})}}.$$

Given the equilibrium outcomes, we next examine comparative statics with respect to t and K_H.

[10]Since $\frac{P'(Q)}{g_x(X,m)} = \frac{m}{m+1+\epsilon}$ from (3.3), the *RMC* condition (4.5) is also expressed as $\frac{P(Q)-r}{g(X,m)-c-t} = \frac{n}{m+1+\epsilon}$, which is the same as (3.8).

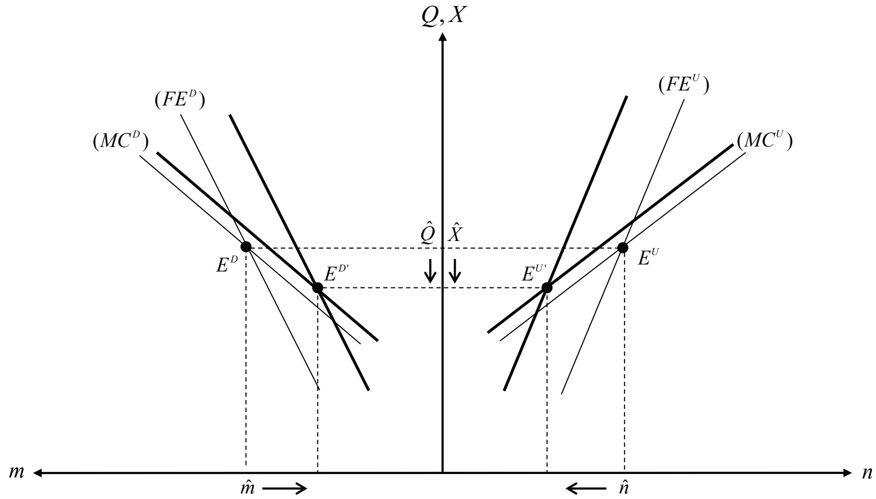

FIGURE 4.3 – Effects of an increase in t

Effect of a change in tariff rate: First we consider the effect of a change in tariff rate t. Observe that t only appears in (4.3) and (4.4). Hence, only MC^U and FE^U are affected by a change in t. From (4.3) and (4.4) it follows that as t increases, n must decrease for any given X and both MC^U and FE^U curves shift to the left; however this shift is greater for FE^U than MC^U. Consequently, as illustrated in Figure 4.3, n must decrease for any given X and both $\hat{X}$ and $\hat{n}$ decline. Further, since $Q = X$, a decline in X implies a decline in Q, which successively induces changes in MC^D and FE^D. From (4.1) and (4.2), as Q decreases, m must decrease and both MC^D and FE^D curves shift to the right; however this shift is greater for FE^D than MC^D. As a result, both $\hat{Q}$ and $\hat{n}$ decline.

Recall from section 3, a tariff lowers equilibrium outputs $(\hat{Q}, \hat{X})$ and raises equilibrium prices $(\hat{P}, \hat{r})$ even when the numbers of Home and Foreign firms are exogenously given. Here, a tariff discourages entry in both sectors of production $\hat{m}, \hat{n}$. This effect lowers outputs and raises prices even further.

It is important to emphasize that trade policy has a crucial impact not only on Foreign firms, but also on Home firms through "firm-colocation" effects. In vertical specialization, Home firms' output and Foreign firms' input are *complements*. Thus, when tariff on intermediate input from Foreign discourages entry of Foreign firms, it also discourages entry of Home firms ($\frac{\partial \hat{m}}{\partial t} < 0, \frac{\partial \hat{n}}{\partial t} < 0$). Note the firm colocation effect occurs only in vertical specialization. If we consider horizontal specialization where Home and Foreign firms' outputs are *substitutes*, a "firm-delocation" effect arises: when tariff on final good from Foreign discourages entry of Foreign firms, it encourages entry of Home firms ($\frac{\partial \hat{m}}{\partial t} > 0, \frac{\partial \hat{n}}{\partial t} < 0$).[11]

[11]For example, Bagwell and Staiger (2012a, b) study long-run effects of trade policy in which two countries trade a homogeneous final good. The markets are segmented and firms compete in a Cournot fashion, whereby two-way trade occurs in a homogeneous good. In the long-run setup with fixed entry costs, they show that higher tariff increases the number of firms in the importing country and decreases the number of firms in the exporting country.

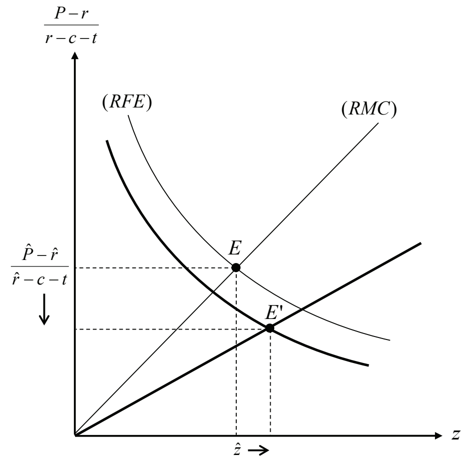

FIGURE 4.4 – Effects of an increase in t

Figure 4.4 illustrates an impact of an increase in t in terms of ***RMC*** and ***RFE*** given by (4.5) and (4.6). As t increases, both ***RMC*** and ***RFE*** curves shift down in $(z, \frac{P-r}{r-c-t})$ space, whereby $\hat{z}$ increases but $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ decreases in long-run equilibrium. The fact that $\hat{z}$ increases with t implies that, although both $\hat{m}$ and $\hat{n}$ are lowered by an increase in t, $\hat{m}$ declines relatively more than $\hat{n}$.[12]

The following lemma summarizes some important comparative statics results that arise from Figures 4.3 and 4.4.

Lemma 4.1

(i) *For a given entry cost K_i, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are decreasing in t; while the final-good price $\hat{P}$ and input-price $\hat{r}$ are increasing in t; i.e., $\partial\hat{Q}/\partial t = \partial\hat{X}/\partial t < 0$, $\partial\hat{P}/\partial t > 0$, and $\partial\hat{r}/\partial t > 0$*

(ii) *For a given entry cost K_i, the number of firms $\hat{m}$, $\hat{n}$ is decreasing in t and the market thickness $\hat{z} = \hat{n}/\hat{m}$ is increasing in t; i.e., $\partial\hat{m}/\partial t < 0$, $\partial\hat{n}/\partial t < 0$ and $\partial\hat{z}/\partial t > 0$.*

(iii) *Let $r^* \equiv \hat{r} - t$ denote the price received by a Foreign firm. Then, there exists $\epsilon^* \in (0,1)$ such that*

$$\frac{\mathrm{d}r^*}{\mathrm{d}t} \lesseqqgtr 0 \quad \Leftrightarrow \quad \frac{\mathrm{d}\hat{r}}{\mathrm{d}t} \lesseqqgtr 1 \quad \Leftrightarrow \quad \epsilon^* \gtreqqless 0.$$

Lemma 4.1 (iii) says that an increase in tariff improves the terms-of-trade, i.e., lowers r^*, if and only if the demand is concave. Recall that when the market structure is exogenous, tariff reduces r^* for all logconcave demand functions ($\epsilon \geq -1$). When the market structure is endogenous, in contrast, tariff reduces r^* only for concave demand functions ($\epsilon \geq \epsilon^*$). This suggests that terms-of-trade improvement is less likely with endogenous market structure. The reasoning goes

[12]Note that if we consider horizontal specialization, it follows from the firm delocation effect that $\frac{\partial\hat{z}}{\partial t} < 0$.

as follows. Differentiating the implicit terms-of-trade $r^* = \hat{r} - t$ and using $\frac{\partial \hat{X}}{\partial t} = \hat{n}\frac{\partial \hat{x}}{\partial t} + \hat{x}\frac{\partial \hat{n}}{\partial t}$,

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m})\hat{n}\frac{\partial \hat{x}}{\partial t} + g_x(\hat{X}, \hat{m})\hat{x}\frac{\partial \hat{n}}{\partial t} + g_m(\hat{X}, \hat{m})\frac{\partial \hat{m}}{\partial t} - 1.$$

Note when the market structure is exogenous, the second and third terms are absent. In other words, when the market structure is endogenous, tariff gives rises to additional adjustments through the exit of Home and Foreign firms. Further substituting $\frac{\partial \hat{m}}{\partial t}$ and $\frac{\partial \hat{n}}{\partial t}$ in Lemma 4.1(ii), the above expression can be simplified as

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m})\frac{\partial \hat{x}}{\partial t}.$$

Thus, the terms-of-trade improvement occurs ($\frac{\partial r^*}{\partial t} < 0$) if and only if the average imported input of Foreign firms $\hat{x}$ increases by tariff ($\frac{\partial \hat{x}}{\partial t} > 0$). Although this is less likely to occur at first glance, we find that whether the average outputs $\hat{q}, \hat{x}$ decrease by tariff depends on the elasticity of slope of demand $\epsilon = \frac{QP''(Q)}{P'(Q)}$ in our model:

$$\frac{\partial \hat{q}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless 0, \quad \frac{\partial \hat{x}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless \epsilon^*.$$

Intuitively, while an increase in t decreases aggregate outputs $\hat{Q}, \hat{X}$, it also discourages entry of firms $\hat{m}, \hat{n}$, which reduces the degree of competition. Consequently, surviving firms might find it profitable to increase their outputs, which is caused by the exit of rival firms. More generally, our model suggests that the decrease in aggregate outputs is largely accounted for by the decrease in the numbers of firms $\hat{m}, \hat{n}$, whereas net changes in the average outputs $\hat{q}, \hat{x}$ are ambiguous.[13]

Effect of a change in entry cost: Recall that we examine comparative statics with respect to the number of Foreign firms n (in addition to t) in the short run. In the long run, however, since the number of firms is an endogenous variable, we cannot conduct these comparative statics. A natural candidate of an exogenous variable that shapes the numbers of Home and Foreign firms m, n (and the relative thickness of markets $\frac{n}{m}$) would then be firms' entry costs, K_H and K_F. Thus we consider the effect of a change in these entry costs. Note these costs can be interpreted as "competition policy" broadly defined, or policies in general – as well as other institutional features of an economy – that make it difficult to start a business. While we focus on the effect of Home's entry cost K_H, the effect of Foreign's entry cost K_F is qualitatively similar. These comparative statics allow us to show that the optimal tariff can affect market thickness as well, but the thickness is still constrained by the limits given by K_H and K_F in the next subsection.

Observe that K_H only appears in (4.2). Hence, only FE^D is affected by a change in K_H. From (4.2) it follows that as K_H increases, m must decrease for any given Q and FE^D curves shift to

[13] This finding is similar with that of Arkolakis et al. (2008), who find that an increase in transport cost decreases the export volume mainly through the numbers of varieties in the model of monopolistic competition and free entry. (Note in the short run where the numbers of firms m, n are fixed, we have that $\frac{\partial \hat{q}}{\partial t} < 0$ and $\frac{\partial \hat{x}}{\partial t} < 0$ for any ϵ.)

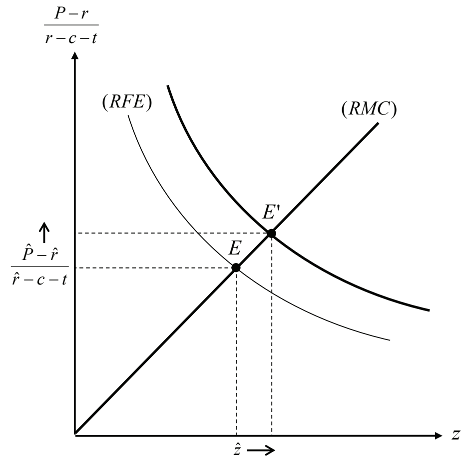

FIGURE 4.5 – Effects of an increase in K_H

the right. Consequently, using a similar diagram in Figure 4.3, we find that m must decrease for any given Q and both $\hat{Q}$ and $\hat{m}$ decline. Further, since $Q = X$, a decline in Q implies a decline in X, which successively induces changes in MC^U and FE^U. From (4.3) and (4.4), as X decreases, m must decrease and both MC^U and FE^U curves shift to the left; however this shift is greater for FE^U than MC^U. As a result, both $\hat{X}$ and $\hat{m}$ decline. Note importantly that, as in trade policy, competition policy also has the "firm-colocation" effects: when it is difficult to start a business in Home, this discourages entry of Foreign firms as well as Home firms. In that sense, there is a complementarity between competition policy and trade policy.

Figure 4.5 illustrates an impact of an increase in K_H in terms of ***RMC*** and ***RFE***. As K_H increases, it follows from (4.5) and (4.6) that only the ***RFE*** curve shifts up in $(z, \frac{P-r}{r-c-t})$ space, whereby both $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ increase in long-run equilibrium. As in the case of t, the fact that $\hat{z}$ increases with K_H implies that, although both $\hat{m}$ and $\hat{n}$ are lowered by an increase in K_H, $\hat{m}$ declines relatively more than $\hat{n}$.

The following lemma summarizes some important comparative statics results with respect to K_H.

Lemma 4.2

(i) *For a given tariff rate t and Foreign entry cost K_F, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are decreasing in K_H; while the final-good price $\hat{P}$ and input price $\hat{r}$ are increasing in K_H; i.e., $\partial\hat{Q}/\partial K_H = \partial\hat{X}/\partial K_H < 0$, $\partial\hat{P}/\partial K_H > 0$, $\partial\hat{r}/\partial K_H > 0$.*

(ii) *For a given tariff rate t and Foreign entry cost K_F, the number of firms $\hat{m}, \hat{n}$ is decreasing in K_H and the market thickness $\hat{z}$ is increasing in K_H; i.e., $\partial\hat{m}/\partial K_H < 0$, $\partial\hat{n}/\partial K_H < 0$ and $\partial\hat{z}/\partial K_H > 0$.*

4.2 Tariffs

In the first stage, the Home government chooses a tariff rate t to maximize Home welfare (W_H), taking the output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$, the price vector $(\hat{P}, \hat{r})$ and the number vector $(\hat{m}, \hat{n})$ as given. As profits are zero under free entry, Home welfare effectively consists of consumer surplus and tariff revenues only. Thus, in the SPNE of Stage 1 subgame, W_H is given by

$$W_H \equiv \underbrace{\left[\int_0^{\hat{Q}} P(y)\mathrm{d}y - P(\hat{Q})\hat{Q}\right]}_{\text{Consumer surplus}} + \underbrace{t\hat{X}}_{\text{Tariff revenue}} . \tag{4.7}$$

Differentiating W_H with respect to t, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \left(1 - \frac{\partial P(\hat{Q})}{\partial t}\right)\hat{Q} + t\frac{\partial \hat{X}}{\partial t}.$$

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ and solving for t gives the expression for the optimal tariff which is presented later in Proposition 4.1. Since $\frac{\partial \hat{X}}{\partial t} < 0$, the optimal tariff is strictly positive (negative) if and only if $1 - \frac{\partial P(\hat{Q})}{\partial t} > (<) 0$. In the short-run analysis, we argue that tariff induces the welfare loss due to the tariff-induced output reduction but the welfare gain arising from the terms-of-trade improvement. However, the above expression is not directly related to how the terms-of-trade r^* improves by tariff.

To better connect the optimal tariff in the short-run and long-run equilibria, noting that the aggregate Home profit is zero under free entry, i.e., $(P(\hat{Q}) - \hat{r})\hat{Q} = \hat{m}K_H$ in the SPNE, we have $P(\hat{Q})\hat{Q} = \hat{r}\hat{Q} + \hat{m}K_H$. Substituting this equality into (4.7) yields

$$W_H = \int_0^{\hat{Q}} P(y)\mathrm{d}y - g(\hat{X}, \hat{m})\hat{Q} - \hat{m}K_H + t\hat{X}. \tag{4.8}$$

The expression (4.8) implies that Home welfare is total surplus defined as gross benefit less the sum of production cost and entry cost (from Home's perspectives). Further, using $\hat{r} - t = r^*$ and simplifying (4.8), we have that

$$W_H = \int_0^{\hat{Q}} P(y)\mathrm{d}y - r^*\hat{X} - \hat{m}K_H.$$

Note this expression is similar to that in the short run except for the extra term $\hat{m}K_H$: in the long-run, $\hat{m}$ entering Home firms pay the entry cost K_H and Home welfare takes into account the total entry cost $\hat{m}K_H$. Noting $\hat{Q} = \hat{X}$ and differentiating this W_H with respect to t, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - r^*)\frac{\partial \hat{Q}}{\partial t} - \frac{\partial r^*}{\partial t}\hat{X} - \frac{\partial \hat{m}}{\partial t}K_H.$$

As in the short run, the first term captures the welfare loss due to the tariff-induced output reduction ($\frac{\partial \hat{Q}}{\partial t} < 0$), and the second term captures the welfare gain arising from the terms-of-trade improvement ($\frac{\partial r^*}{\partial t} < 0$). In contrast to the short run, however, the third term captures the welfare loss due to the tariff-induced reduction of Home firms ($\frac{\partial \hat{m}}{\partial t} < 0$), which arises only in the long run. In addition, the terms-of-trade improvement does not always occur for all logconcave demand functions and tariff reduces r^* only for concave demand functions ($\frac{\partial r^*}{\partial t} < 0$ if and only if $\epsilon > \epsilon^*$).

Using the expression for $\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1$, we can express $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ as follows:

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} - \frac{\partial \hat{m}}{\partial t}K_H + t\frac{\partial \hat{X}}{\partial t}. \tag{4.9}$$

Following the previous section, we first focus on the optimal tariff. Noting that $\frac{\partial \hat{X}}{\partial t} < 0$ in (4.9), the optimal tariff is strictly positive (negative) if and only if

$$(P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} - \frac{\partial \hat{m}}{\partial t}K_H > (<)0. \tag{4.10}$$

Contrary to Section 3, the sign of the optimal tariff cannot be argued by the numbers of Home and Foreign firms $\hat{m}, \hat{n}$ as these numbers are not parameters in the long run. Since $\frac{\partial \hat{Q}}{\partial t} = \hat{m}\frac{\partial \hat{q}}{\partial t} + \hat{q}\frac{\partial \hat{m}}{\partial t}$ and $(P(\hat{Q}) - \hat{r})\hat{q} - K_H = 0$ under free entry, condition (4.10) is rewritten as

$$(P(\hat{Q}) - \hat{r})\hat{m}\frac{\partial \hat{q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} > (<)0. \tag{4.11}$$

It is important to note that (4.11) is exactly the same as (3.7) since $\hat{Q} = m\hat{q}$ and hence $\frac{\partial \hat{Q}}{\partial t} = m\frac{\partial \hat{q}}{\partial t}$ in the short run. This implies that the sign of the optimal tariff does not depend on whether the profits are positive or zero. Instead, it depends on the signs of $\frac{\partial \hat{q}}{\partial t}$ and $\frac{\partial \hat{r}}{\partial t} - 1 = \frac{\partial r^*}{\partial t}$, which are in turn crucially influenced by whether the market structure is exogenous or endogenous.[14]

The comparative statics in Section 4.1 tell us that whether $\frac{\partial \hat{q}}{\partial t}$ and $\frac{\partial r^*}{\partial t}$ increase depends on the curvature of the demand function ϵ. Together with (4.11), the sign of the optimal tariff also depends on ϵ. In particular, the optimal tariff is positive for concave demand functions ($\epsilon \geq \epsilon^*$) as both $\frac{\partial \hat{q}}{\partial t} > 0$ and $\frac{\partial r^*}{\partial t} > 0$, whereas the optimal tariff is negative for convex demand functions ($\epsilon \leq 0$) as both $\frac{\partial \hat{q}}{\partial t} < 0$ and $\frac{\partial r^*}{\partial t} < 0$ (note if demand is linear ($\epsilon = 0$), $\frac{\partial \hat{q}}{\partial t} = 0$ and $\frac{\partial r^*}{\partial t} > 0$ and thus the optimal tariff is always negative for any $\hat{Q}, \hat{X}, \hat{m}, \hat{n}$.) Further, from (4.9) and $\frac{\partial \hat{m}}{\partial t} < 0$, the optimal tariff t is increasing in the entry cost K_H, which comes from a complementarity between trade policy and competition policy through the firm colocation effect. This suggests that a reduction of import tariffs for component has its largest effect on welfare when accompanied by liberalization of entry in the domestic final-good market.

[14] More specifically, $\frac{\partial \hat{q}}{\partial t} < 0$ and $\frac{\partial r^*}{\partial t} < 0$ in the exogenous market structure, whereas $\frac{\partial \hat{q}}{\partial t} \gtreqless 0$ and $\frac{\partial r^*}{\partial t} \gtreqless 0$ in the endogenous market structure.

To see this from another angle, suppose that the Home government can choose not only tariff rate t but also entry cost K_H to maximize Home welfare in the SPNE of Stage 1 subgame. Then, differentiating (4.8) with respect to K_H gives

$$\frac{\mathrm{d}W_H}{\mathrm{d}K_H} = (P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial K_H} - \frac{\partial \hat{r}}{\partial K_H}\hat{Q} - \hat{m} - \frac{\partial \hat{m}}{\partial K_H}K_H + t\frac{\partial \hat{X}}{\partial K_H}. \tag{4.12}$$

Noting that $\frac{\partial \hat{m}}{\partial K_H} < 0$ in (4.12), the optimal entry cost is strictly positive if and only if

$$(P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial K_H} - \frac{\partial \hat{r}}{\partial K_H}\hat{Q} - \hat{m} + t\frac{\partial \hat{X}}{\partial K_H} < 0.$$

Since $\frac{\partial \hat{Q}}{\partial K_H} < 0$ and $\frac{\partial \hat{r}}{\partial K_H} > 0$, the optimal competition policy is positive if the optimal tariff is positive (i.e., $t > 0$). Further from (4.12) and $\frac{\partial \hat{X}}{\partial K_H} < 0$, the optimal entry cost K_H is increasing in the tariff rate t. Proposition 4.1 presents these findings and provides a sharper characterization.

Proposition 4.1 *Let $t(K_H)$ denote the optimal tariff. At $t = t(K_H)$, the following holds:*

$$t = -\hat{Q}P'(\hat{Q})\left(\frac{2(\hat{m}+\hat{n})\epsilon + (\epsilon+1)(\epsilon-2)}{4\hat{m}\hat{n}}\right), \tag{4.13}$$

where $\hat{Q}$ is the aggregate output evaluated at $t = t(K_H)$. Furthermore,

(i) *There exists $\epsilon^{**} \in (0,1)$ such that*

$$t(K_H) \gtreqless 0 \iff \epsilon \gtreqless \epsilon^{**}.$$

(ii) *$t(K_H)$ is monotonically increasing in K_H.*

Recall that there exists a *negative* relationship between $\frac{n}{m}$ and t in the short-run equilibrium: the optimal tariff t is higher, the thicker is the Home final-good market relative to Foreign input market (i.e., lower $\frac{n}{m}$), as depicted in Figure 3.1. In the long-run equilibrium, however, this relationship is overturned and our model predicts that there exists a *positive* relationship between $\frac{n}{m}$ and t. As noted above, in the long run where firms can freely enter and exit, the market thickness is constrained by the limits of the entry costs, K_H and K_F. Further it follows from Lemma 4.2 that the greater K_H makes it more difficult to start a business not only for Home firms but also for Foreign firms through the firm colocation effect, whereby the market thickness $\hat{z} = \frac{\hat{n}}{\hat{m}}$ is higher. At the same time, since there is a complementarity between competition policy and trade policy, the greater K_H also induces the higher optimal tariff t, as seen in Proposition 4.1. Combining these two observations establishes that the optimal tariff t is higher, the thinner is the Home final-good market (i.e., higher $\frac{n}{m}$), as depicted in Figure 4.6. This finding suggests that reduction of import tariff for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

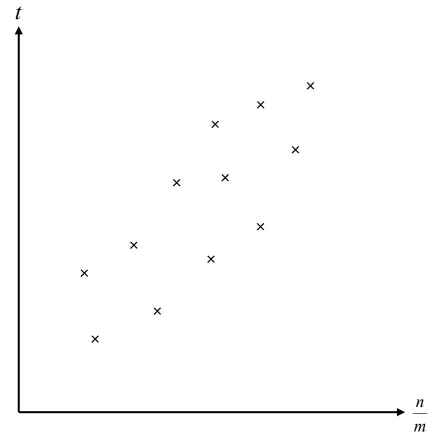

FIGURE 4.6 – Optimal tariff in long run

What should we make of the fact that (a) demand curvature matters for the sign of the optimal tariff and (b) the relationship between market thickness and tariff differs between the two cases – endogenous and exogenous market structures? Our reading of the literature suggests that, in terms of the dependence of optimal policy on demand curvature, our results have a similar flavor to some of the existing results in the trade literature. For example, the classic result that the sign of the optimal tariff in the presence of a foreign monopoly depends on whether there is incomplete pass-through, which in turn depends on whether the demand curve is flatter than the marginal revenue curve (Brander and Spencer, 1984a,b; Helpman and Krugman, 1989, Chapter 4). Concerning the difference in results between endogenous and exogenous market structures, our finding is in the line with Horstmann and Markusen (1986) and Venables (1985), who have shown that in the single-stage oligopoly models, entry can alter optimal trade policy due to firm-delocation effects. This point has also recently been made by Etro (2011) and Bagwell and Staiger (2012a, b) in the contexts of strategic trade policy and trade agreements respectively. We do not necessarily view (b) as a shortcoming. Depending on the industry characteristics, such as industry-specific fixed costs or stability of demand, some industries fit an exogenous market structure description better, while for some other industries with fluid entry and volatile demand, an endogenous market structure is more apt.

As an illustrative example, consider again the following class of inverse demand functions: $P(Q) = a - Q^b, b > 0$. Observe that both $\hat{m}$ and $\hat{n}$ are eliminated from the expression of the optimal tariff (4.13) since these numbers can be explicitly solved by applying this specific demand function to the free entry conditions, (4.2) and (4.4). Solving (4.2), (4.4) and (4.13) yields

$$t = \text{to be added}$$

Note the property of the optimal tariff in Proposition 4.1 – in particular a positive relationship between t and K_H – holds for this specific demand function.

5 Conclusion

To be added

Appendix

A Proofs for Section 3

A.1 Equivalence between Assumptions 1 and 1'

The assumption $Q(P)$ is logconcave implies

$$\frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{\mathrm{d}\ln Q(P)}{\mathrm{d}P}\right] = \frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{Q'(P)}{Q(P)}\right]$$
$$= \frac{Q(P)\cdot Q''(P) - [Q'(P)]^2}{[Q(P)]^2} \leq 0,$$

which can be expressed as

$$\frac{Q(P)Q''(P)}{[Q'(P)]^2} \leq 1. \tag{A.1}$$

Differentiating $P = P(Q(P))$ with respect to P, we get

$$1 = P'(Q(P))Q'(P).$$

Differentiating this once again with respect to P gives

$$0 = P''[Q'(P)]^2 + P'Q''(P).$$

Rewriting this equation, we get

$$\frac{Q''(P)}{[Q'(P)]^2} = -\frac{P''}{P'}.$$

Substituting this relationship into (A.1), we find that

$$-\frac{QP''(Q)}{P'(Q)} \leq 1,$$

which implies $P'(Q) + QP''(Q) \leq 0$. □

A.2 Proof of Lemma 3.1

(i) Differentiating (3.5) with respect to n, rearranging and using (3.5) subsequently, we get

$$\frac{\partial \hat{X}}{\partial n} = -\frac{g(\hat{X}) - c - t}{(n+1)g'(\hat{X}) + \hat{X}g''(\hat{X})} = \frac{\hat{x}}{n+1+\epsilon}.$$

Note $\hat{Q} = \hat{X}$ implies that $\frac{\partial \hat{Q}}{\partial n} = \frac{\partial \hat{X}}{\partial n}$. Since $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$, we get

$$\frac{\partial \hat{q}}{\partial n} = \frac{\hat{q}}{n(n+1+\epsilon)}, \qquad \frac{\partial \hat{x}}{\partial n} = -\frac{(n+\epsilon)\hat{x}}{n(n+1+\epsilon)}.$$

Using the expression for $\frac{\partial \hat{X}}{\partial n}$ we get

$$\frac{\partial \hat{r}}{\partial n} = \frac{\partial g(\hat{X})}{\partial n} = g'(\hat{X})\frac{\partial \hat{X}}{\partial n} = \frac{\hat{x}g'(\hat{X})}{n+1+\epsilon} = \frac{\hat{x}P'(\hat{Q})(m+1+\epsilon)}{m(n+1+\epsilon)},$$
$$\frac{\partial \hat{P}}{\partial n} = \frac{\partial P(\hat{Q})}{\partial n} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial n} = \frac{\hat{x}P'(\hat{Q})}{n+1+\epsilon}.$$

The results follow from noticing that $P'(\hat{Q}) < 0$, $m+1+\epsilon > 0$ and $n+1+\epsilon > 0$.

(ii) Differentiating (3.5) with respect to t, we get

$$\frac{\partial \hat{X}}{\partial t} = \frac{n}{g'(\hat{X})(n+1+\epsilon)} = \frac{mn}{P'(\hat{Q})(n+1+\epsilon)(m+1+\epsilon)}.$$

Note $\hat{Q} = \hat{X}$ implies that $\frac{\partial \hat{Q}}{\partial t} = \frac{\partial \hat{X}}{\partial t}$. Since $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$, we get

$$\frac{\partial \hat{q}}{\partial t} = \frac{n}{P'(\hat{Q})(m+1+\epsilon)(n+1+\epsilon)}, \qquad \frac{\partial \hat{x}}{\partial t} = \frac{m}{P'(\hat{Q})(m+1+\epsilon)(n+1+\epsilon)}.$$

Using the expression for $\frac{\partial \hat{X}}{\partial t}$ we get

$$\frac{\partial \hat{r}}{\partial t} = g'(\hat{X})\frac{\partial \hat{X}}{\partial t} = \frac{n}{n+1+\epsilon}, \qquad \frac{\partial \hat{P}}{\partial t} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{mn}{(m+1+\epsilon)(n+1+\epsilon)}.$$

The results follow from noticing that $P'(\hat{X}) < 0$, $n+1+\epsilon > 0$ and $n+1+\epsilon > 0$.

(iii) We have that

$$\frac{\mathrm{d}r^*}{\mathrm{d}t} = \frac{\mathrm{d}\hat{r}}{\mathrm{d}t} - 1 = -\frac{1+\epsilon}{n+1+\epsilon}.$$

The claim follows from observing that $n+1+\epsilon > 0$.

Although we have focused on comparative statics with respect to n, it is straightforward to examine comparative statics with respect to n. From (3.1), we have that

$$\frac{\partial \hat{Q}}{\partial m} = -\frac{P(\hat{Q}) - r}{(m+1)P'(\hat{Q}) + \hat{Q}P''(\hat{Q})} = \frac{\hat{q}}{m+1+\epsilon}.$$

Since $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$, we get

$$\frac{\partial \hat{q}}{\partial m} = -\frac{(1+\epsilon)\hat{q}}{m(m+1+\epsilon)}, \qquad \frac{\partial \hat{x}}{\partial m} = \frac{\hat{x}}{m(m+1+\epsilon)}.$$

Regarding the prices, note in particular that the input price r depends on m as well as X (see (3.2)). While we apply the short-hand definition $r \equiv g(X)$ for the short-run analysis (since we mainly focus on comparative statics with respect to n), we need to explicitly define $r \equiv g(X, m)$

when we conduct comparative statics with respect to m. Thus

$$\frac{\partial \hat{r}}{\partial m} = \frac{\partial g(\hat{X}, m)}{\partial m} = g_x(\hat{X}, m)\frac{\partial \hat{X}}{\partial m} + g_m(\hat{X}, m) = 0,$$
$$\frac{\partial \hat{P}}{\partial m} = \frac{\partial P(\hat{Q})}{\partial m} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial m} = \frac{\hat{q}P'(\hat{Q})}{m+1+\epsilon},$$

where

$$g_x(X, m) \equiv \frac{\partial g(X, m)}{\partial X}, \quad g_m(X, m) \equiv \frac{\partial g(X, m)}{\partial m}.$$

The results follow from noticing that $P'(\hat{Q}) < 0$ and $m + 1 + \epsilon > 0$. □

A.2 Proof of Proposition 3.1

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ in (3.6) and rearranging yields (3.9). Concerning the properties of the optimal tariff $t = t(n)$, since (i) directly follows from (3.9), we focus on (ii) below.

First we show that t is decreasing in n. Differentiating $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ with respect to n gives:

$$\frac{\mathrm{d}t}{\mathrm{d}n} = -\frac{\frac{\partial^2 W_H}{\partial n \partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.$$

For the class of inverse demand functions that satisfy Assumptions 1 and 2, W_H is strictly concave in t so that the second-order condition is satisfied, i.e., $\frac{\partial^2 W_H}{\partial t^2} < 0$. Then it follows that

$$\mathrm{sgn}\frac{\mathrm{d}t}{\mathrm{d}n} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial n \partial t}. \tag{A.2}$$

Using (3.8), rewrite $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ in (3.6) as

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \frac{n}{m+1+\epsilon}(\hat{r} - c - t)\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} + t\frac{\partial \hat{X}}{\partial t}. \tag{A.3}$$

Differentiating (A.3) with respect to n gives

$$\frac{\partial^2 W_H}{\partial n \partial t} = \frac{1}{m+1+\epsilon}(\hat{r} - c - t)\frac{\partial \hat{Q}}{\partial t} < 0.$$

Since $\frac{\partial^2 W_H}{\partial n \partial t} < 0$, (A.2) implies that $\frac{\mathrm{d}t}{\mathrm{d}n} < 0$. Similarly, we can show that $t = t(m, n)$ is increasing in m by replacing n with m in (A.2) and by differentiating (A.3) with respect to m.

Next we show for future reference that the above also holds by directly differentiating t with respect to n and m. To show $\frac{\mathrm{d}t}{\mathrm{d}n} < 0$, using $P'(Q) = \frac{m}{m+1+\epsilon}g'(X)$ from (3.3), rewrite the optimal tariff in (3.9) as $t = -\hat{X}g'(\hat{X})\Phi$ where $\Phi \equiv \frac{(1+\epsilon)(m+1+\epsilon)-n}{mn}$. Differentiating this t with respect to n,

$$\begin{aligned}\frac{\mathrm{d}t}{\mathrm{d}n} &= -\left[g'(\hat{X}) + \hat{X}g''(\hat{X})\right]\frac{\mathrm{d}\hat{X}}{\mathrm{d}n}\Phi - \hat{X}g'(\hat{X})\frac{\mathrm{d}\Phi}{\mathrm{d}n} \\ &= -g'(\hat{X})(1+\epsilon)\Phi\left(\frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{X}}{\partial t}\frac{\mathrm{d}t}{\mathrm{d}n}\right) + \hat{X}g'(\hat{X})\left(\frac{1+\epsilon}{n^2}\right).\end{aligned}$$

Substituting $\frac{\partial \hat{X}}{\partial n}$ and $\frac{\partial \hat{X}}{\partial t}$ from Lemma 3.1 and solving for $\frac{\mathrm{d}t}{\mathrm{d}n}$ yields

$$\frac{\mathrm{d}t}{\mathrm{d}n} = -(1+\epsilon)\left(-\frac{\hat{X}g'(\hat{X})}{n}\right)\left[\frac{m+2+\epsilon}{mn+(1+\epsilon)(2+\epsilon)(m+1+\epsilon)}\right].$$

The claim follows from noting that $g'(X) < 0$ and $1+\epsilon > 0$. Following the similar steps, we get

$$\frac{\mathrm{d}t}{\mathrm{d}m} = \left(-\frac{\hat{Q}P'(\hat{Q})}{m}\right)\left[\frac{n+1+\epsilon}{mn+(1+\epsilon)(2+\epsilon)(m+1+\epsilon)}\right].$$

The claim follows from noting that $P'(Q) < 0$. □

A.3 Proof of Proposition 3.2

We first show that the impact of n on Π_F is decomposed into the competition effect and tariff-reduction effect. Differentiating $\Pi_F = (\hat{r} - c - t)\hat{X}$ with respect to n,

$$\begin{aligned}\frac{\mathrm{d}\Pi_F}{\mathrm{d}n} &= (\hat{r} - c - t)\frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{r}}{\partial n}\hat{X} - \frac{\mathrm{d}t}{\mathrm{d}n} \\ &= -\hat{x}g'(\hat{X})\frac{\partial \hat{X}}{\partial n} + n\hat{x}g'(\hat{X})\frac{\partial \hat{X}}{\partial n} - \frac{\mathrm{d}t}{\mathrm{d}n} \\ &= (n-1)\hat{x}g'(\hat{X})\frac{\partial \hat{X}}{\partial n} - \frac{\mathrm{d}t}{\mathrm{d}n}.\end{aligned}$$

Next we show that the size effect can dominate the competition effect. Differentiating $\Pi_F = (\hat{r} - c - t)\hat{X} = -\frac{\hat{X}^2 g'(\hat{X})}{n}$ with respect to n gives

$$\frac{\mathrm{d}\Pi_F}{\mathrm{d}n} = \frac{\hat{X}^2 g'(\hat{X})}{n^2}\left[1 - (2+\epsilon)\delta\right], \tag{A.4}$$

where $\delta \equiv \frac{n}{\hat{X}}\frac{\mathrm{d}\hat{X}}{\mathrm{d}n} = \frac{n}{\hat{X}}\left(\frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{X}}{\partial t}\frac{\mathrm{d}t}{\mathrm{d}n}\right)$. Substituting $\frac{\partial \hat{X}}{\partial n}$, $\frac{\partial \hat{X}}{\partial t}$ from Lemma 3.1 and $\frac{\mathrm{d}t}{\mathrm{d}n}$ from Proposition 3.1(ii), we get

$$\delta = \frac{1}{n+1+\epsilon}\left[\frac{mn + (m+1+\epsilon)(1+\epsilon)(2+\epsilon) + n(1+\epsilon)(m+2+\epsilon)}{mn + (m+1+\epsilon)(1+\epsilon)(2+\epsilon)}\right].$$

Since $\frac{\hat{X}^2 g'(\hat{X})}{n^2} < 0$ in (A.4), $\frac{\mathrm{d}\Pi_F}{\mathrm{d}n} > 0 \Leftrightarrow \delta > \frac{1}{2+\epsilon}$. Evaluating $\lim_{m\to\infty}\delta$ and solving the last inequality for n establishes the result. $\square$

B Proofs of Section 4

B.1 Proof of Lemma 4.1

Let a dot represent proportional rates of change (e.g. $\dot{Q} \equiv \frac{Q'}{Q}$) and totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$(n+1+\epsilon)\dot{X} = \left(\frac{n+1+\epsilon}{m+1+\epsilon}\right)\dot{m} + \dot{n} + \frac{mnt}{(m+1+\epsilon)QP'(Q)}\dot{t}, \tag{B.1}$$

$$(2+\epsilon)\dot{Q} = 2\dot{m}, \tag{B.2}$$

$$(2+\epsilon)\dot{X} = \left(\frac{1+\epsilon}{m+1+\epsilon}\right)\dot{m} + 2\dot{n}, \tag{B.3}$$

where K_H and K_F hold constant. (B.1), (B.2) and (B.3) are three equations that have three unknowns $\dot{X}(=\dot{Q})$, $\dot{m}$ and $\dot{n}$, which can be solved explicitly as a function of $\dot{t}$:

$$\dot{X} = \left(\frac{4}{\Omega}\right)\frac{mnt}{QP'(Q)}\dot{t},$$

$$\dot{m} = \left(\frac{2(2+\epsilon)}{\Omega}\right)\frac{mnt}{QP'(Q)}\dot{t},$$

$$\dot{n} = \left(\frac{2+\epsilon}{\Omega}\right)\left(\frac{2m+1+\epsilon}{m+1+\epsilon}\right)\frac{mnt}{QP'(Q)}\dot{t},$$

where $\Omega \equiv (2m+\epsilon)(2n+\epsilon) - (2+\epsilon) > 0$ for $m > 1$ and $n > 1$. Evaluated at $X = \hat{X}, m = \hat{m}, n = \hat{n}$, we have that

$$\frac{\partial \hat{X}}{\partial t} = \left(\frac{4}{\hat{\Omega}}\right)\frac{\hat{m}\hat{n}}{P'(\hat{Q})} < 0, \tag{B.4}$$

$$\frac{\partial \hat{m}}{\partial t} = \left(\frac{2(2+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}^2\hat{n}}{\hat{Q}P'(\hat{Q})} < 0, \tag{B.5}$$

$$\frac{\partial \hat{n}}{\partial t} = \left(\frac{2+\epsilon}{\hat{\Omega}}\right)\left(\frac{(2\hat{m}+1+\epsilon)\hat{n}^2}{\hat{X}g'(\hat{X})}\right) < 0. \tag{B.6}$$

Further, since $\hat{Q} = \hat{X}$, $\hat{Q} = \hat{m}\hat{q}$ and $\hat{X} = \hat{n}\hat{x}$, we have $\frac{\partial \hat{Q}}{\partial t} = \hat{m}\frac{\partial \hat{q}}{\partial t} + \hat{q}\frac{\partial \hat{m}}{\partial t}$ and $\frac{\partial \hat{X}}{\partial t} = \hat{n}\frac{\partial \hat{x}}{\partial t} + \hat{x}\frac{\partial \hat{n}}{\partial t}$, and using (B.4), (B.5) and (B.6) yields

$$\frac{\partial \hat{q}}{\partial t} = -\frac{2\hat{n}\epsilon}{\hat{\Omega}P'(\hat{Q})}, \tag{B.7}$$

$$\frac{\partial \hat{x}}{\partial t} = -\frac{2\hat{m}\epsilon + (\epsilon+1)(\epsilon-2)}{\hat{\Omega}g_x(\hat{X})}, \tag{B.8}$$

which suggests that

$$\frac{\partial \hat{q}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless 0,$$
$$\frac{\partial \hat{x}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless \epsilon^*,$$

where $\epsilon^* \in (0,1)$ satisfies $2\hat{m}\epsilon^* + (\epsilon^*+1)(\epsilon^*-2) = 0$. Using the expressions of $\frac{\partial \hat{X}}{\partial t}$ and $\frac{\partial \hat{m}}{\partial t}$ in (B.4) and (B.5), we also have that

$$\frac{\partial \hat{P}}{\partial t} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{4\hat{m}\hat{n}}{\hat{\Omega}} > 0,$$
$$\frac{\partial \hat{r}}{\partial t} = g_x(\hat{X},\hat{m})\frac{\partial \hat{X}}{\partial t} + g_m(\hat{X},\hat{m})\frac{\partial \hat{m}}{\partial t} = \frac{2\hat{n}(2\hat{m}+\epsilon)}{\hat{\Omega}} > 0.$$

Next, differentiating $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial t}$ and $\frac{\partial \hat{m}}{\partial t}$ in (B.4) and (B.5), we have that

$$\frac{\partial \hat{z}}{\partial t} = -\left(\frac{(1+\epsilon)(2+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}k}{\hat{Q}P'(\hat{Q})} > 0,$$
$$\frac{\partial\left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}\right)}{\partial t} = \left(\frac{(1+\epsilon)(2+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}k}{\hat{X}g_x(\hat{X})} < 0.$$

Finally, using the expression of $\frac{\partial \hat{r}}{\partial t}$, it directly follows that

$$\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1 = -\frac{2\hat{m}\epsilon + (\epsilon+1)(\epsilon-2)}{\hat{\Omega}}. \tag{B.9}$$

Comparing (B.8) and (B.9) suggests that

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X},\hat{m})\frac{\partial \hat{x}}{\partial t}.$$

The claim that $\frac{\partial r^*}{\partial t} \lesseqgtr 0 \Leftrightarrow \frac{\partial \hat{x}}{\partial t} \gtreqless 0$ follows from noting that $g_x(X,m) < 0$. □

B.2 Proof of Lemma 4.2

Using a dot representation once again (e.g. $\dot{Q} \equiv \frac{Q'}{Q}$) and totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$(n+1+\epsilon)\dot{X} = \left(\frac{n+1+\epsilon}{m+1+\epsilon}\right)\dot{m} + \dot{n}, \tag{B.10}$$
$$(2+\epsilon)\dot{X} = 2\dot{m} + \dot{K}_H, \tag{B.11}$$
$$(2+\epsilon)\dot{X} = \left(\frac{1+\epsilon}{m+1+\epsilon}\right)\dot{m} + 2\dot{n}, \tag{B.12}$$

where t and K_F hold constant. (B.10), (B.11) and (B.12) are the three equations that have the three unknowns $\dot{X}(=\dot{Q})$, $\dot{m}$ and $\dot{n}$, which can be solved explicitly as a function of $\dot{K}_H$:

$$\dot{X} = -\left(\frac{2n+1+\epsilon}{\Omega}\right)\dot{K}_H,$$
$$\dot{m} = -\left(\frac{(m+1+\epsilon)(2n+\epsilon)}{\Omega}\right)\dot{K}_H,$$
$$\dot{n} = -\left(\frac{n+1+\epsilon}{\Omega}\right)\dot{K}_H,$$

where Ω is exactly the same as before. Evaluated at $X=\hat{X}, m=\hat{m}, n=\hat{n}$, we have that

$$\frac{\partial \hat{X}}{\partial K_H} = -\left(\frac{2\hat{n}+1+\epsilon}{\hat{\Omega}}\right)\frac{\hat{X}}{K_H} < 0, \tag{B.13}$$
$$\frac{\partial \hat{m}}{\partial K_H} = -\left(\frac{(\hat{m}+1+\epsilon)(2\hat{n}+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}}{K_H} < 0, \tag{B.14}$$
$$\frac{\partial \hat{n}}{\partial K_H} = -\left(\frac{\hat{n}+1+\epsilon}{\hat{\Omega}}\right)\frac{\hat{n}}{K_H} < 0. \tag{B.15}$$

Further, since $\hat{Q}=\hat{X}$, $\hat{Q}=\hat{m}\hat{q}$ and $\hat{X}=\hat{n}\hat{x}$, we have $\frac{\partial \hat{Q}}{\partial t} = \hat{m}\frac{\partial \hat{q}}{\partial t} + \hat{q}\frac{\partial \hat{m}}{\partial t}$ and $\frac{\partial \hat{X}}{\partial t} = \hat{n}\frac{\partial \hat{x}}{\partial t} + \hat{x}\frac{\partial \hat{n}}{\partial t}$, and using (B.13), (B.14) and (B.15) yields

$$\frac{\partial \hat{q}}{\partial K_H} = \left(\frac{(\hat{m}+\epsilon)(2\hat{n}+\epsilon)-1}{\hat{\Omega}}\right)\frac{\hat{q}}{K_H} > 0,$$
$$\frac{\partial \hat{x}}{\partial K_H} = -\left(\frac{\hat{n}}{\hat{\Omega}}\right)\frac{\hat{x}}{K_H} < 0.$$

Using the expressions of $\frac{\partial \hat{X}}{\partial K_H}$ and $\frac{\partial \hat{m}}{\partial K_H}$ in (B.13) and (B.14), we also have that

$$\frac{\partial \hat{P}}{\partial K_H} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial K_H} = -\left(\frac{2\hat{n}+1+\epsilon}{\hat{\Omega}}\right)\frac{\hat{Q}P'(\hat{Q})}{K_H} > 0,$$
$$\frac{\partial \hat{r}}{\partial K_H} = g_x(\hat{X},\hat{m})\frac{\partial \hat{X}}{\partial K_H} + g_m(\hat{X},\hat{m})\frac{\partial \hat{m}}{\partial K_H} = -\left(\frac{1}{\hat{\Omega}}\right)\frac{\hat{X}g'(\hat{X},\hat{m})}{K_H} > 0.$$

Next, differentiating $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial K_H}$ and $\frac{\partial \hat{m}}{\partial K_H}$ in (B.13) and (B.14), we have that

$$\frac{\partial \hat{z}}{\partial K_H} = \left(\frac{(\hat{m}+\epsilon)(2\hat{n}+\epsilon)+(\hat{n}-1)}{\hat{\Omega}}\right)\frac{\hat{z}}{K_H} > 0,$$
$$\frac{\partial\left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}\right)}{\partial K_H} = \left(\frac{(\hat{m}-1)(\hat{n}+\epsilon)+(\hat{m}\hat{n}-1)}{\hat{\Omega}}\right)\frac{1}{\hat{z}K_F} > 0.$$

□

B.2 Proof of Proposition 4.1

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ in (4.9) and rearranging yields (4.13). Note from (4.13) that

$$t \gtreqless 0 \iff \epsilon \gtreqless \epsilon^{**},$$

where $\epsilon^{**} \in (0,1) < \epsilon^{*}$ satisfies $2(\hat{m}+\hat{n})\epsilon^{**} + (\epsilon^{**}+1)(\epsilon^{**}-2) = 0$. Concerning the properties of the optimal tariff $t = t(K_H)$, since (i) directly follows from (4.13), we focus on (ii) below.

We first show that t is increasing in K_H. Differentiating $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ with respect to K_H, we get

$$\frac{\mathrm{d}t}{\mathrm{d}K_H} = -\frac{\frac{\partial^2 W_H}{\partial K_H \partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.$$

For the class of inverse demand functions that satisfy Assumptions 1 and 2, W_H is strictly concave in t so that the second-order condition is satisfied, i.e., $\frac{\partial^2 W_H}{\partial t^2} < 0$. Then it follows that

$$\operatorname{sgn}\frac{\mathrm{d}t}{\mathrm{d}K_H} = \operatorname{sgn}\frac{\partial^2 W_H}{\partial K_H \partial t}. \tag{B.16}$$

Further differentiating (4.9) with respect to K_H gives

$$\frac{\partial^2 W_H}{\partial K_H \partial t} = -\frac{\partial \hat{m}}{\partial t} > 0.$$

From (B.16) and the above expression, it follows that $\frac{\mathrm{d}t}{\mathrm{d}K_H} > 0$. □

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